LINEAR PROGRAMMING (Q 11, PAPER 2)

2001

11 (a) Using graph paper, illustrate the set of points (that simultaneously satisfy the three inequalities:

$$y \geq 2$$
$$x + 2y \leq 8$$
$$5x + y \geq -5.$$

(b) Houses are to be built on 9 hectares of land.
Two types of houses, bungalows and semi-detached houses, are possible.

Each bungalow occupies one fifth of a hectare.
Each semi-detached house occupies one tenth of a hectare.

The cost of building a bungalow is IR£80 000.
The cost of building a semi-detached house is IR£50 000.
The total cost of building the houses cannot be greater than IR£4 million.

(i) Taking x to represent the number of bungalows and y to represent the number of semi-detached houses, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The profit on each bungalow is IR£10 000. The profit on each semi-detached house is IR£7000. How many of each type of house should be built so as to maximise profit?

SOLUTION

11 (a)

DRAWING LINEAR INEQUALITIES

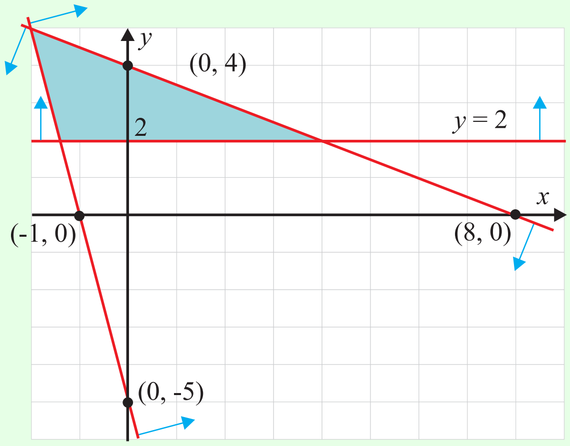
STEPS
1. Graph the equation of the line first by finding the x and y intercepts.
2. Take a test point like (0, 0) and substitute it into the inequality.
3. If you get a true result, shade in the side of the line containing (0, 0).
If you get a false result, shade in the side **not** containing (0, 0).

NOTE: If the line passes through (0, 0) then choose another point like (1, 0).

Graph $x + 2y \leq 8$.
1. Draw $x + 2y = 8$. Intercepts: (0, 4), (8, 0)
2. Test with (0, 0): $(0) + 2(0) \leq 8 \Rightarrow 0 \leq 8$. This is true.
3. Shade in the side of the line that contains (0, 0).

Graph $5x + y \geq -5$.
1. Draw $5x + y = -5$. Intercepts: (0, –5), (–1, 0)
2. Test with (0, 0): $5(0) + (0) \geq -5 \Rightarrow 0 \geq -5$. This is true.
3. Shade in the side of the line that contains (0, 0).

Graph $y \geq 2$.
Draw a horizontal line through $y = 2$ and shade above the line.



Draw the lines. The blue arrows indicate the side of the line for which the inequality is true. These regions all overlap in the region where the three lines intersect. Shade in this region. The points in this region simultaneously satisfy the three inequalities.

11 (b) **MAXIMISING AND MINIMISING PROBLEMS**

STEPS

1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of bungalows
Let y = Number of semi-detached houses

2.

	Bungalows	Houses	Restriction
Space	$\frac{1}{5}x$	$\frac{1}{10}y$	9
Cost	$80000x$	$50000y$	4000000

Space inequality: $\frac{1}{5}x + \frac{1}{10}y \leq 9 \Rightarrow 2x + y \leq 90$

Cost inequality: $80000x + 50000y \leq 4000000 \Rightarrow 8x + 5y \leq 400$

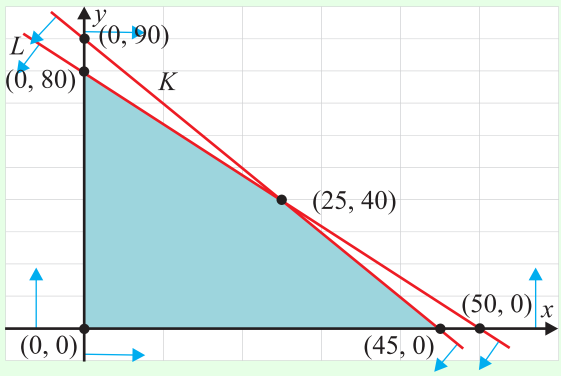
As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

3. Plot the four inequalities.
Graph $2x + y \leq 90$. Draw the line $2x + y = 90$. Call it K.

Intercepts: (0, 90), (45, 0). Test with (0, 0) $\Rightarrow 2(0) + (0) = 0 \leq 90$. This is true.
Shade the side of the line that contains (0, 0).

Graph $8x + 5y \leq 400$. Draw the line $8x + 5y = 400$. Call it L.

Intercepts: (0, 80), (50, 0). Test with (0, 0) $\Rightarrow 8(0) + 5(0) = 0 \leq 400$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 80) and (45, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$\begin{aligned} 2x+y &= 90.....(\mathbf{1}) \quad (\times -4) \\ 8x+5y &= 400...(\mathbf{2}) \end{aligned}$$

$$\begin{aligned} -8x-4y &= -360 \\ 8x+5y &= 400 \\ \hline y &= 40 \end{aligned}$$

Substitute y = 40 back into Eqn. (**1**).

$\Rightarrow 2x+(40)=90 \Rightarrow 2x=50 \Rightarrow x=25$

Therefore (25, 40) is the final vertex of the region.

5. Profit = 10000x + 7000y is the function to be maximised.

	10000x + 7000y	**Income**
(0, 0)	10000(0) + 7000(0)	€0
(0, 80)	10000(0) + 7000(80)	€**560,000**
(25, 40)	10000(25) + 7000(40)	€530,000
(45, 0)	10000(45) + 7000(0)	€450,000

Therefore, 0 bungalows and 80 semi-detached houses give the maximum profit.

ANSWERS

11 (b) (i) $2x+y \le 90,\ 8x+5y \le 400$

11 (b) (ii) x = 0, y = 80