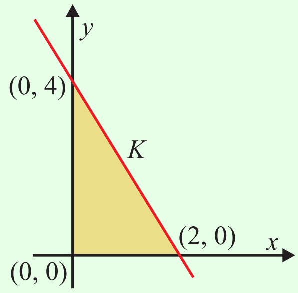
LINEAR PROGRAMMING (Q 11, PAPER 2)

2000

11 (a) The line K passes through the points (2, 0) and (0, 4).

(i) Find the equation of the line K.

(ii) Write down three inequalities which define the shaded region in the diagram.



(b) Two types of machines, type A and type B, can be purchased for a new factory. Each machine of type A costs IR£1600. Each machine of type B costs IR£800. The purchase of the machines can cost, at most, IR£27,200.

Each machine of type A needs 90 m^2 of floor space in the factory. Each machine of type B needs 54 m^2 of floor space.

The maximum amount of floor space available for the machines is 1620 m^2.

(i) If x represents the number of machines of type A and y represents the number of machines of type B, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The daily income from the use of each machine of type A is IR£75. The daily income from the use of each machine of type B machine is IR£42. How many of each type of machine should be purchased so as to maximise daily income?

(iii) What is the maximum daily income?

SOLUTION

11 (a) (i)

(2, 0) (0, 4)
$\downarrow\downarrow$ $\downarrow\downarrow$
$x_1\ y_1$ $x_2\ y_2$

Slope: $m=\frac{y_2-y_1}{x_2-x_1}$ 3

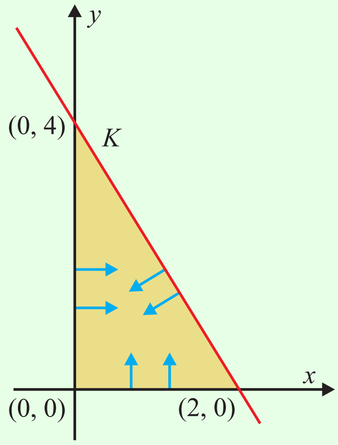
Equation of a line: $y-y_1=m(x-x_1)$ 4

Slope $m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-0}{0-2}=\frac{4}{-2}=-2$

Equation of line K: $y-0=-2(x-2)\Rightarrow y=-2x+4$

$\therefore 2x+y=4$

11 (a) (ii)



VERTICAL LINES: $\geq$: $\rightarrow$ (Right) $\leq$: $\leftarrow$ (Left)

HORIZONTAL LINES: $\geq$: $\uparrow$ (Above) $\leq$: $\downarrow$ (Below)

Inequality 1: Above the x-axis $\Rightarrow x \geq 0$

Inequality 2: Right of the y-axis $\Rightarrow y \geq 0$

STEPS

2. Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.

3. The side of the line with (0, 0) obeys the inequality found in Step **2**. The other side is the opposite to the inequality found in Step **2**.

Inequality 3:

2. Substitute (0, 0) into $K \Rightarrow 2(0)+(0)=0 \leq 4$

3. The indicated region is on the same side as (0, 0).

Therefore, $2x+y \leq 4$ is the inequality of the indicated region.

Three inequalities: $2x+y \leq 4,\ x \geq 0,\ y \geq 0$

11 (b)

MAXIMISING AND MINIMISING PROBLEMS

STEPS

1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of type A machines
Let y = Number of type B machines

2.

	Type A	Type B	Restriction
Cost	$1600x$	$800y$	27200
Floor space	$90x$	$54y$	1620

Cost inequality: $1600x+800y \leq 27200 \Rightarrow 2x+y \leq 34$

Floor space inequality: $90x+54y \leq 1620 \Rightarrow 5x+3y \leq 90$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

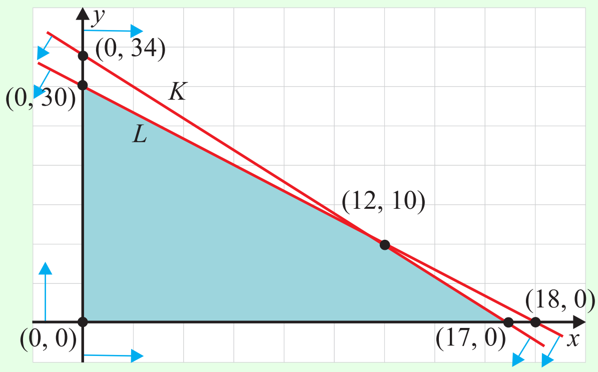
3. Plot the four inequalities.

Graph $2x + y \le 34$. Draw the line 2*x* + *y* = 34. Call it *K*.

Intercepts: (0, 34), (17, 0). Test with (0, 0) $\Rightarrow 2(0) + (0) = 0 \le 34$. This is true.
Shade the side of the line that contains (0, 0).

Graph $5x + 3y \le 90$. Draw the line 5*x* + 3*y* = 90. Call it *L*.

Intercepts: (0, 30), (18, 0). Test with (0, 0) $\Rightarrow 5(0) + 3(0) = 0 \le 90$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 30) and (17, 0).
The only one you need to work out simultaneously is where the lines *K* and *L* intersect.

$$2x + y = 34 \ldots(\mathbf{1}) \quad (\times -3)$$
$$5x + 3y = 90 \ldots(\mathbf{2})$$

$$-6x - 3y = -102$$
$$5x + 3y = 90$$
$$-x = -12 \Rightarrow x = 12$$

Substitute *x* = 12 back into Eqn. (**1**).

$\Rightarrow 2(12) + y = 34 \Rightarrow y = 34 - 24 = 10$

Therefore (12, 10) is the final vertex of the region.

5. Income = 75*x* + 42*y* is the function to be maximised.

	75*x* + 42*y*	**Income**
(0, 0)	75(0) + 42(0)	€0
(0, 30)	75(0) + 42(30)	€1260
(12, 10)	75(12) + 42(10)	€**1320**
(17, 0)	75(17) + 42(0)	€1275

Therefore, 12 type A machines and 10 type B machines give the maximum income.

Answers

11 (b) (i) $2x + y \le 34,\ 5x + 3y \le 90$

11 (b) (ii) A = 12, B = 10

11 (b) (iii) £1320