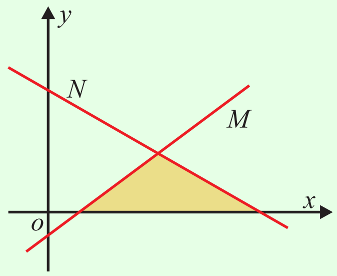
Linear Programming (Q 11, Paper 2)

1999

11 (a) The equation of the line M is $x - y - 1 = 0$
and the equation of the line N is $x + 2y - 6 = 0$.
Write down the three inequalities which define the triangular region indicated in the diagram.



(b) A company uses small trucks and large trucks to transport its products in crates. The crates are all of the same size.

On a certain day 10 truck drivers at most are available. Each truck requires one driver only.

Small trucks take 10 minutes each to load and large trucks take 30 minutes each to load. The total loading time must not be more than 3 hours. Only one truck can be loaded at a time.

(i) If x represents the number of small trucks used and y represents the number of large trucks used, write down two inequalities in x and y.

Illustrate these on graph paper.

(ii) Each small truck carries 30 crates and each large truck carries 70 crates. How many of each type of truck should be used to maximize the number of crates to be transported that day?

Solution

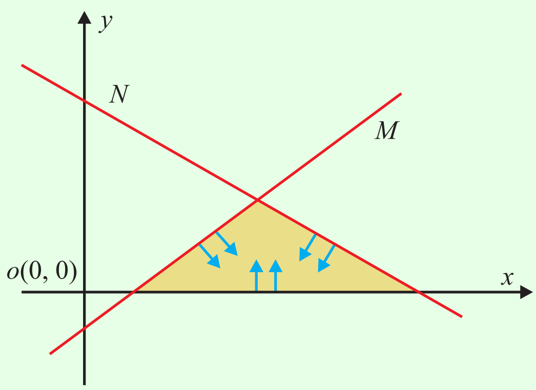
11 (a)

Equation of M: $x - y - 1 = 0$

Test with (0, 0): $(0) - (0) - 1 = -1 \le 0$

Shaded side is on the opposite side as (0, 0).

$\therefore x - y - 1 \ge 0$



Equation of N: $x + 2y - 6 = 0$

Test with (0, 0): $(0) + 2(0) - 6 = -6 \le 0$

Shaded side is on the same side as (0, 0).

$x + 2y - 6 \le 0$

Above the x-axis: $y = 0$

Shaded side above this line.

$\therefore y \ge 0$

Horizontal Lines:	$\ge$: ↑ (Above) $\le$: ↓ (Below)

11 (b) **Maximising and Minimising Problems**

Steps
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of small trucks
Let y = Number of large trucks

2.

	Small trucks	Large trucks	Restriction
Drivers	x	y	10
Loading time	$10x$	$30y$	180

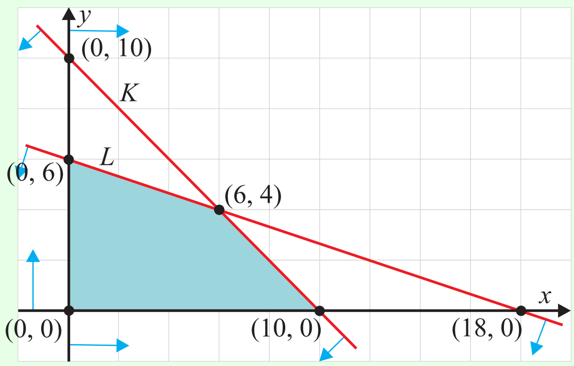
Drivers inequality: $x+y\leq 10$

Loading time inequality: $10x+30y\leq 180 \Rightarrow x+3y\leq 18$

As always, there are two inequalities that are obvious: $x\geq 0$ and $y\geq 0$.

3. Plot the four inequalities.
Graph $x+y\leq 10$. Draw the line $x+y=10$. Call it K.
Intercepts: (0, 10), (10, 0). Test with (0, 0) $\Rightarrow (0)+(0)=0\leq 10$. This is true.
Shade the side of the line that contains (0, 0).

Graph $x+3y\leq 18$. Draw the line $x+3y=18$. Call it L.
Intercepts: (0, 6), (18, 0). Test with (0, 0) $\Rightarrow (0)+3(0)=0\leq 18$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 6) and (10, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$\begin{aligned} x+y &= 10 \ldots(\mathbf{1}) \quad (\times -1) \\ x+3y &= 18 \ldots(\mathbf{2}) \end{aligned}$$

$$\begin{aligned} -x-y &= -10 \\ x+3y &= 18 \\ \hline 2y &= 8 \Rightarrow y = 4 \end{aligned}$$

Substitute y = 4 back into Eqn. (**1**).

$\Rightarrow x+(4) = 10 \Rightarrow x = 6$

Therefore (6, 4) is the final vertex of the region.

5. Crates cargo = $30x + 70y$ is the function to be maximised.

	$30x + 70y$	**Cargo**
(0, 0)	30(0) + 70(0)	0
(0, 6)	30(0) + 70(6)	420
(6, 4)	30(6) + 70(4)	**460**
(10, 0)	30(10) + 70(0)	300

Therefore, 6 small trucks and 4 large trucks give the maximum cargo.

ANSWERS:

11 (b) (i) $x+y \le 10,\ x+3y \le 18$

11 (b) (ii) $x = 6$, $y = 4$