LINEAR PROGRAMMING (Q 11, PAPER 2)

1998

11 (a) Write down the coordinates of two points on the line $2x + 3y = 18$.

On a diagram, illustrate the set of points (x, y) that satisfy simultaneously the three inequalities

$$2x + 3y \leq 18$$
$$x \geq 3$$
$$y \geq 2.$$

(b) A company produces two products, A and B.

Each unit of the two products must be processed on two assembly lines, the red line and the blue line, for a certain length of time.

Each unit of A requires 3 hours on the red line and 1 hour on the blue line.
Each unit of B requires 1 hour on the red line and 2 hours on the blue line.

Each week, the maximum time available on the red line is 60 hours and the maximum time available on the blue line is 40 hours.

(i) If x represents the number of units of A produced in a week and y represents the number of units of B produced in a week, write down two inequalities in x and y. Illustrate these on graph paper.

(ii) The profit made on each unit of A is twice the profit made on each unit of B. How many units of each product must be manufactured in a week so as to maximise the profit?

(iii) If the maximum profit that can be made in a week is IR£1980, calculate the profit made on each unit of A and on each unit of B.

SOLUTION

11 (a)

DRAWING LINEAR INEQUALITIES

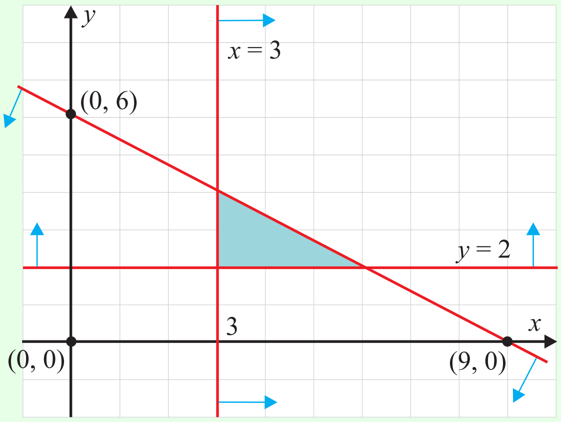
STEPS
1. Graph the equation of the line first by finding the x and y intercepts.
2. Take a test point like (0, 0) and substitute it into the inequality.
3. If you get a true result, shade in the side of the line containing (0, 0).
If you get a false result, shade in the side **not** containing (0, 0).

NOTE: If the line passes through (0, 0) then choose another point like (1, 0).

Graph $2x + 3y \leq 18$.
1. Draw $2x + 3y = 18$. Intercepts: (0, 6), (9, 0)
2. Test with (0, 0): $2(0) + 3(0) \leq 18 \Rightarrow 0 \leq 18$. This is true.
3. Shade in the side of the line that contains (0, 0).

Graph $x \geq 3$.
Draw a vertical line through x = 3 and shade to the right of the line.

Graph $y \geq 2$.
Draw a horizontal line through y = 2 and shade above the line.



Draw the lines. The blue arrows indicate the side of the line for which the inequality is true. These regions all overlap in the region where the three lines intersect. Shade in this region. The points in this region simultaneously satisfy the three inequalities.

11 (b) **MAXIMISING AND MINIMISING PROBLEMS**

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of units of A
Let y = Number of units of B

2.

	A	B	Restriction
Red line	$3x$	y	60
Blue line	x	$2y$	40

Red line inequality: $3x + y \leq 60$

Blue line inequality: $x + 2y \leq 40$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

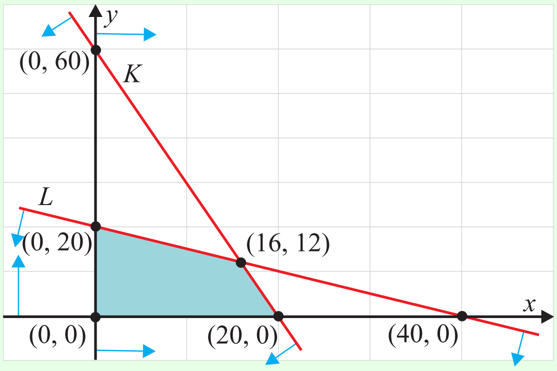
3. Plot the four inequalities.
Graph $3x + y \leq 60$. Draw the line $3x + y = 60$. Call it K.

Intercepts: (0, 60), (20, 0). Test with (0, 0) $\Rightarrow 3(0) + (0) = 0 \leq 60$. This is true.
Shade the side of the line that contains (0, 0).

Graph $x+2y \le 40$. Draw the line $x + 2y = 40$. Call it L.

Intercepts: (0, 20), (40, 0). Test with (0, 0) $\Rightarrow (0)+2(0)=0 \le 40$.

This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 20) and (20, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$3x+y=60\ldots\mathbf{(1)} \quad (\times -2)$$
$$x+2y=40\ldots\mathbf{(2)}$$

$$-6x-2y=-120$$
$$x+2y=40$$
$$-5x \qquad = -80 \Rightarrow x=16$$

Substitute x = 16 back into Eqn. (**2**).

$\Rightarrow (16)+2y=40 \Rightarrow 2y=24 \Rightarrow y=12$

Therefore (16, 12) is the final vertex of the region.

5. Profit = $2x + y$ is the function to be maximised.

	$2x + y$	**Profit**
(0, 0)	2(0) + (0)	0
(0, 20)	2(0) + (20)	20
(16, 12)	2(16) + (12)	**44**
(20, 0)	2(20) + (0)	40

Therefore, 16 units of A and 12 units of B give the maximum profit.

Answers:

11 (b) (i) $3x+y \le 60,\ x+2y \le 40$

11 (b) (ii) 16 of A and 12 of B

11 (b) (iii)
A profit of £44 was generated from producing 16 units of A and 12 units of B using the fact that £2 profit was made of each unit of A against £1 profit for each unit of B.

Profit on each unit of B $= \dfrac{£1980}{44} = £45$

Profit on each unit of A $= £90$