Linear Programming (Q 11, Paper 2)

1997

11 (a) On one diagram, illustrate the set of points (x, y) that satisfy the three inequalities

$$x + y \leq 7$$
$$2x + y \geq 8$$
$$x \geq 0.$$

(b) A factory, which manufactures television sets makes two types of set - a wide screen model and a standard model.

In any week, 500 sets at most can be manufactured.

Each wide screen model costs IR£200 to produce. Each standard model costs IR£150 to produce. Total weekly production costs must not be greater than IR£90,000.

(i) If the factory manufactures x of the wide screen model and y of the standard model, write down two inequalities in x and y and illustrate these on graph paper.

(ii) If the profit on a wide screen model is IR£100 and the profit on a standard model is IR£70, how many of each type of set should be manufactured in order to maximise profit?

Solution

11 (a)

Drawing Linear Inequalities

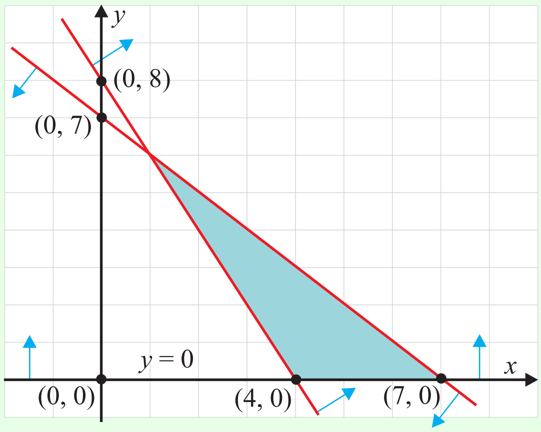
Steps
1. Graph the equation of the line first by finding the x and y intercepts.
2. Take a test point like (0, 0) and substitute it into the inequality.
3. If you get a true result, shade in the side of the line containing (0, 0).
If you get a false result, shade in the side **not** containing (0, 0).

Note: If the line passes through (0, 0) then choose another point like (1, 0).

Graph $x + y \leq 7$.
1. Draw $x + y = 7$. Intercepts: (0, 7), (7, 0)
2. Test with (0, 0): $(0) + (0) \leq 7 \Rightarrow 0 \leq 7$. This is true.
3. Shade in the side of the line that contains (0, 0).

Graph $2x + y \geq 8$.
1. Draw $2x + y = 8$. Intercepts: (0, 8), (4, 0)
2. Test with (0, 0): $2(0) + (0) \geq 8 \Rightarrow 0 \geq 8$. This is false.
3. Shade in the opposite side to the line that contains (0, 0).

Graph $y \geq 0$.
Draw a horizontal line through $y = 0$ (the x-axis) and shade above the line.



Draw the lines. The blue arrows indicate the side of the line for which the inequality is true. These regions all overlap in the region where the three lines intersect. Shade in this region. The points in this region simultaneously satisfy the three inequalities.

11 (b) **MAXIMISING AND MINIMISING PROBLEMS**

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of wide screen models
Let y = Number of standard models

2.

	Wide screen	Standard	Restriction
Output	x	y	500
Cost	$200x$	$150y$	90000

Output inequality: $x + y \leq 500$

Costs inequality: $200x + 150y \leq 90000 \Rightarrow 4x + 3y \leq 1800$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

3. Plot the four inequalities.

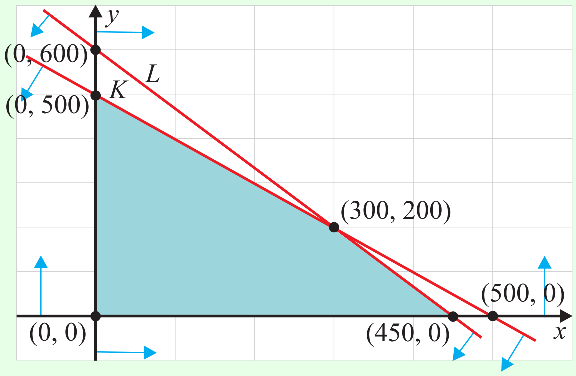
Graph $x + y \leq 500$. Draw the line $x + y = 500$. Call it K.

Intercepts: (0, 500), (500, 0). Test with (0, 0) $\Rightarrow (0) + (0) = 0 \leq 500$. This is true.
Shade the side of the line that contains (0, 0).

Graph $4x+3y \le 1800$. Draw the line $4x + 3y = 1800$. Call it L.

Intercepts: (0, 600), (450, 0). Test with (0, 0) $\Rightarrow 4(0)+3(0)=0 \le 1800$.

This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 500) and (450, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$x+y = 500 \quad \ldots..(\mathbf{1}) \quad (\times -3)$$
$$4x+3y = 1800 \quad \ldots(\mathbf{2})$$

$$-3x-3y = -1500$$
$$4x+3y = 1800$$
$$x = 300$$

Substitute $x = 300$ back into Eqn. (**1**).

$\Rightarrow (300)+y=500 \Rightarrow y=200$

Therefore (300, 200) is the final vertex of the region.

5. Profit = $100x + 70y$ is the function to be maximised.

	$100x + 70y$	**Profit**
(0, 0)	100(0) + 70(0)	£0
(0, 500)	100(0) + 70(500)	£35,000
(300, 200)	100(300) + 70(200)	£44,000
(450, 0)	100(450) + 70(0)	£45,000

Therefore, 450 wide screen models and 0 standard models give the maximum profit.

Answers

11 (b) (i) $x+y \le 500,\ 4x+3y \le 1800$

11 (b) (ii) $x = 450, y = 0$