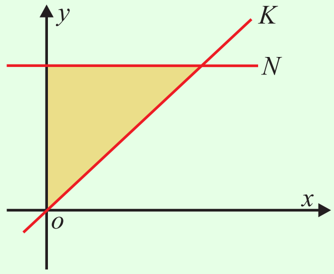
LINEAR PROGRAMMING (Q 11, PAPER 2)

1996

11 (a) The equation of the line K is $y - x = 0$ and the equation of the line N is $y - 4 = 0$.



(i) Write down the three inequalities which define the triangular region indicated in the diagram.

(ii) In a diagram, illustrate the set of points (x, y) that satisfy $y - 4 \geq 0$, $y - x \leq 0$ and $x - 6 \leq 0$.

(b) A property developer wishes to construct a business centre consisting of shops and offices. The floor space required for each shop is 60 m^2 and for each office is 20 m^2. The total floor space for the business centre cannot exceed 960 m^2.

The construction of each shop takes 5 working days to complete and each office 3 working days to complete. The developer has at most 120 working days to complete the construction.

(i) If the developer constructs x shops and y offices, write two inequalities in x and y and illustrate these on graph paper.

(ii) If the rental charge is IR£200 per m^2 for a shop and IR£140 per m^2 for an office, how many of each type should be built so as to maximize the developer's rental income? Find this maximum rental income.

(iii) If each shop provides 7 jobs and each office 3 jobs, write an expression in x and y for the total number of jobs to be provided. How many of each type should be built so as to maximize the number of jobs?

SOLUTION

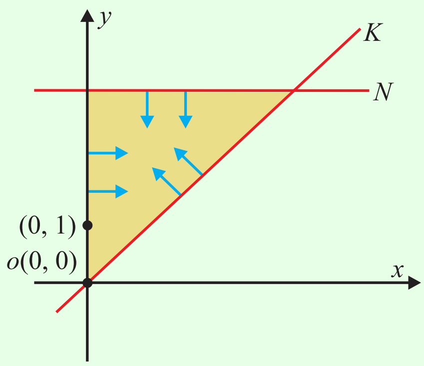
11 (a) (i)

Equation of K: $y - x = 0$

Test with (0, 1): $(1) - (0) = 1 \geq 0$

Shaded side is on the same side as (0, 1).

$\therefore y - x \geq 0$



Equation of N: $y - 4 = 0$

Test with (0, 0): $(0) - 4 = -4 \leq 0$

Shaded side is on the same side as (0, 0).

$\therefore y - 4 \leq 0$

y-axis: $x = 0$

Shaded side is right of the line.

$\therefore x \geq 0$

VERTICAL LINES:	$\geq$: $\rightarrow$ (Right)
	$\leq$: $\leftarrow$ (Left)

11 (a) (ii)

Graph $y-4 \geq 0$.

Draw $y-4=0 \Rightarrow y=4$.

Draw a line through $y = 4$ and shade above the line.

HORIZONTAL LINES: $\geq$: ↑ (Above) $\leq$: ↓ (Below)

Graph $y-x \leq 0$.

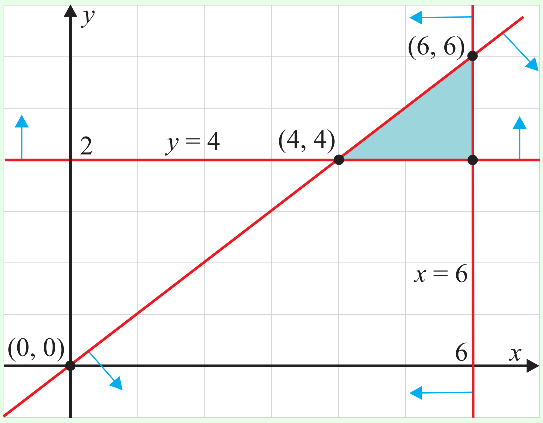
Draw $y = x$. This is a line through (0, 0). It contains points where the first and second co-ordinates are equal like (4, 4) and (6, 6).

Graph $x-6 \leq 0$.

Draw $x-6=0 \Rightarrow x=6$.

Draw a line through $x = 6$ and shade to the left of the line.

VERTICAL LINES: $\geq$: → (Right) $\leq$: ← (Left)



Draw the lines. The blue arrows indicate the side of the line for which the inequality is true. These regions all overlap in the region where the three lines intersect. Shade in this region. The points in this region simultaneously satisfy the three inequalities.

11 (b) **MAXIMISING AND MINIMISING PROBLEMS**

STEPS

1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of shops
Let y = Number of offices

2.

	Shops	Offices	Restriction
Floor space (m^2)	$60x$	$20y$	960
Time (Days)	$5x$	$3y$	120

Adults inequality: $60x+20y \leq 960 \Rightarrow 3x+y \leq 48$

Children inequality: $5x+3y \leq 120$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

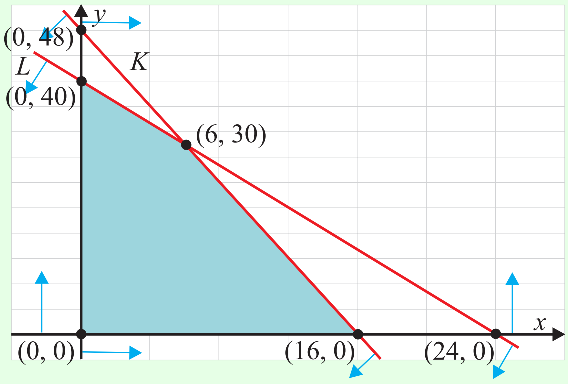
3. Plot the four inequalities.

Graph $3x + y \le 48$. Draw the line $3x + y = 48$. Call it K.

Intercepts: (0, 48), (16, 0). Test with (0, 0) $\Rightarrow 3(0) + (0) = 0 \le 48$. This is true.
Shade the side of the line that contains (0, 0).

Graph $5x + 3y \le 120$. Draw the line $5x + 3y = 120$. Call it L.

Intercepts: (0, 40), (24, 0). Test with (0, 0) $\Rightarrow 5(0) + 3(0) = 0 \le 120$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 40) and (16, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$3x + y = 48 \ldots\ldots(\mathbf{1}) \quad (\times -3)$$
$$5x + 3y = 120 \ldots(\mathbf{2})$$

$$-9x - 3y = -144$$
$$5x + 3y = 120$$
$$-4x = -24 \Rightarrow x = 6$$

Substitute $x = 6$ back into Eqn. (**1**).

$\Rightarrow 3(6) + y = 48 \Rightarrow y = 48 - 18 = 20$

Therefore (6, 20) is the final vertex of the region.

5. Rental income = $200x + 140y$ is the function to be maximised.

	$200x + 140y$	**Income**
(0, 0)	200(0) + 140(0)	£0
(0, 40)	200(0) + 140(40)	£**5600**
(6, 20)	200(6) + 140(20)	£4000
(16, 0)	200(16) + 140(0)	£3200

Therefore, 0 shops and 40 offices give the maximum rental income.

5. No. of jobs = $7x + 3y$ is the function to be maximised.

	$7x + 3y$	**Jobs**
(0, 0)	7(0) + 3(0)	0
(0, 40)	7(0) + 3(40)	**120**
(6, 20)	7(6) + 3(20)	58
(16, 0)	7(16) + 3(0)	112

Therefore, 0 shops and 40 offices give the maximum number of jobs.

Answers

11 (b) (i) $3x + y \leq 48$, $5x + 3y \leq 120$

11 (b) (ii) $x = 0$, $y = 40$; £5600

11 (b) (iii) $x = 0$, $y = 40$