Linear Programming (Q 11, Paper 2)

Lesson No. 3: Practical Linear Programming Problems

2007

11 (b) A developer is planning a holiday complex of cottages and apartments.
Each cottage will accommodate 3 adults and 5 children and each apartment will accommodate 2 adults and 2 children.
The other facilities in the complex are designed for a maximum of 60 adults and a maximum of 80 children.

(i) Taking x as the number of cottages and y as the number of apartments, write down two inequalities in x and y and illustrate these on graph paper.

(ii) If the rental income per night will be €65 for a cottage and €40 for an apartment, how many of each should the developer include in the complex to maximise potential rental income?

(iii) If the construction costs are €200 000 for a cottage and €120 000 for an apartment, how many of each should the developer include in the complex to minimise construction costs?

2006

11 (b) Due to a transport disruption, a bus company is contracted at short notice to carry up to 1500 passengers to complete their journey. Passengers not carried by this company will be carried by a taxi company.

The bus company has available standard buses and mini-buses. Each standard bus carries 60 passengers and each mini-bus carries 30 passengers.

Each bus is operated by one driver and there are at most 30 drivers available.

(i) Taking x as the number of standard buses and y as the number of mini-buses, write down two inequalities in x and y and illustrate them on graph paper.

(ii) The operating profit for the journey is €80 for a standard bus and €50 for a minibus. How many of each type of bus should be used in order to maximise the profit?

(iii) If the bus company paid each driver a bonus for working at short notice, the operating profit for each bus would be reduced by €30. By how much would this decrease the maximum profit available to the company?

2005

11 (b) A manufacturer of garden furniture produces plastic chairs and tables. Each chair requires 2 kg of raw material and each table requires 5 kg of raw material. In any working period the raw material used cannot exceed 800 kg.

Each chair requires 4 minutes of machine time and each table requires 4 minutes of machine time. The total machine time available in any working period is 1000 minutes.

(i) Taking x as the number of chairs and y as the number of tables, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The manufacturer sells each chair for €20 and each table for €40.
How many of each should be produced in each working period to maximise income?

(iii) The manufacturer's costs for each chair are €17 and for each table are €34.70.
Express the profit as a percentage of income, assuming the income has been maximised.

2004

11 (b) A shop-owner displays videos and DVDs in his shop.
Each video requires 720 cm^3 of display space and each DVD requires 360 cm^3 of display space. The available display space cannot exceed 108 000 cm^3. The shopowner buys each video for €6 and each DVD for €8. He does not wish to spend more than €1200.

(i) Taking x as the number of videos and y as the number of DVDs, write down two inequalities in x and y and illustrate these on graph paper.

During a DVD promotion the selling price of a video is €11 and of a DVD is €10. Assuming that the shop-owner can sell all the videos and DVDs,

(ii) how many of each type should he display in order to maximise his income?

(iii) how many of each type should he display in order to maximise his profit?

2003

11 (b) A developer is planning a scheme of holiday homes, consisting of large and small bungalows. Each large bungalow will accommodate 8 people and each small bungalow will accommodate 6 people. The development is not permitted to accommodate more than 216 people. The floor area of each large bungalow is 200 m^2 and the floor area of each small bungalow is 100 m^2. The total floor area of all the bungalows must not exceed 4000 m^2.

(i) Taking x as the number of large bungalows and y as the number of small bungalows, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The expected net annual income from each large bungalow is €14 000 and from each small bungalow is €8000. How many of each type should be built in order to maximise the total expected net annual income?

(iii) The developer decides to build as indicated in part **(ii)**. The cost of building each large bungalow is €110 000 and the cost of building each small bungalow is €85 000. The total cost of the development is equal to the building costs plus €1.58 million. How many years will it take to recoup the total cost of the development?

2002

11 (b) A new ship is being designed. It can have two types of cabin accommodation for passengers — type A cabins and type B cabins.

Each type A cabin accommodates 6 passsengers and each type B cabin accommodates 3 passengers. The maximum number of passengers that the ship can accommodate is 330.

Each type A cabin occupies 50 m^3 of floor space. Each type B cabin occupies 10 m^3 of floor space. The total amount of floor space occupied by cabins cannot exceed 2300 m^3.

(i) Taking x to represent the number of type A cabins and y to represent the number of type B cabins, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The income on each voyage from renting the cabins to passengers is €600 for each type A cabin and €180 for each type B cabin. How many of each type of cabin should the ship have so as to maximise income, assuming that all cabins are rented?

(iii) What is the maximum possible income on each voyage from renting the cabins?

2001

11 (b) Houses are to be built on 9 hectares of land.
Two types of houses, bungalows and semi-detached houses, are possible.

Each bungalow occupies one fifth of a hectare.
Each semi-detached house occupies one tenth of a hectare.

The cost of building a bungalow is IR£80 000.
The cost of building a semi-detached house is IR£50 000.
The total cost of building the houses cannot be greater than IR£4 million.

(i) Taking x to represent the number of bungalows and y to represent the number of semi-detached houses, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The profit on each bungalow is IR£10 000. The profit on each semi-detached house is IR£7000. How many of each type of house should be built so as to maximise profit?

2000

11 (b) Two types of machines, type A and type B, can be purchased for a new factory.
Each machine of type A costs IR£1600. Each machine of type B costs IR£800.
The purchase of the machines can cost, at most, IR£27,200.

Each machine of type A needs 90 m^2 of floor space in the factory.
Each machine of type B needs 54 m^2 of floor space.

The maximum amount of floor space available for the machines is 1620 m^2.

(i) If x represents the number of machines of type A and y represents the number of machines of type B, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The daily income from the use of each machine of type A is IR£75. The daily income from the use of each machine of type B machine is IR£42. How many of each type of machine should be purchased so as to maximise daily income?

(iii) What is the maximum daily income?

1999

11 (b) A company uses small trucks and large trucks to transport its products in crates. The crates are all of the same size.

On a certain day 10 truck drivers at most are available. Each truck requires one driver only.

Small trucks take 10 minutes each to load and large trucks take 30 minutes each to load. The total loading time must not be more than 3 hours. Only one truck can be loaded at a time.

(i) If x represents the number of small trucks used and y represents the number of large trucks used, write down two inequalities in x and y.

Illustrate these on graph paper.

(ii) Each small truck carries 30 crates and each large truck carries 70 crates. How many of each type of truck should be used to maximize the number of crates to be transported that day?

1998

11 (b) A company produces two products, A and B.

Each unit of the two products must be processed on two assembly lines, the red line and the blue line, for a certain length of time.

Each unit of A requires 3 hours on the red line and 1 hour on the blue line.
Each unit of B requires 1 hour on the red line and 2 hours on the blue line.

Each week, the maximum time available on the red line is 60 hours and the maximum time available on the blue line is 40 hours.

(i) If x represents the number of units of A produced in a week and y represents the number of units of B produced in a week, write down two inequalities in x and y. Illustrate these on graph paper.

(ii) The profit made on each unit of A is twice the profit made on each unit of B. How many units of each product must be manufactured in a week so as to maximise the profit?

(iii) If the maximum profit that can be made in a week is IR£1980, calculate the profit made on each unit of A and on each unit of B.

1997

11 (b) A factory, which manufactures television sets makes two types of set - a wide screen model and a standard model.

In any week, 500 sets at most can be manufactured.

Each wide screen model costs IR£200 to produce. Each standard model costs IR£150 to produce. Total weekly production costs must not be greater than IR£90,000.

(i) If the factory manufactures x of the wide screen model and y of the standard model, write down two inequalities in x and y and illustrate these on graph paper.

(ii) If the profit on a wide screen model is IR£100 and the profit on a standard model is IR£70, how many of each type of set should be manufactured in order to maximise profit?

1996

11 (b) A property developer wishes to construct a business centre consisting of shops and offices. The floor space required for each shop is 60 m^2 and for each office is 20 m^2. The total floor space for the business centre cannot exceed 960 m^2.

The construction of each shop takes 5 working days to complete and each office 3 working days to complete. The developer has at most 120 working days to complete the construction.

(i) If the developer constructs x shops and y offices, write two inequalities in x and y and illustrate these on graph paper.

(ii) If the rental charge is IR£200 per m^2 for a shop and IR£140 per m^2 for an office, how many of each type should be built so as to maximize the developer's rental income? Find this maximum rental income.

(iii) If each shop provides 7 jobs and each office 3 jobs, write an expression in x and y for the total number of jobs to be provided. How many of each type should be built so as to maximize the number of jobs?

ANSWERS

2007 11 (b) (i) $3x + 2y \le 60,\ 5x + 2y \le 80$
(ii) 10 cottages, 15 apartments
(iii) 16 cottages and 0 apartments

2006 11 (b) (i) $x + y \le 30,\ 2x + y \le 50$
(ii) $x = 20,\ y = 10$
(iii) €850

2005 11 (b) (i) $2x + 5y \le 800,\ x + y \le 250$
(ii) $x = 150,\ y = 100$
(iii) 14%

2004 11 (b) (i) $2x + y \le 300,\ 3x + 4y \le 600$
(ii) $x = 120,\ y = 60$
(iii) $x = 150,\ y = 0$

2003 11 (b) (i) $4x + 3y \le 108,\ 2x + y \le 40$
(ii) $x = 6,\ y = 28$
(iii) 15

2002 11 (b) (i) $2x + y \le 110,\ 5x + y \le 230$
(ii) $x = 40,\ y = 30$
(iii) €29,400

2001 11 (b) (i) $2x + y \le 90,\ 8x + 5y \le 400$
(ii) $x = 0,\ y = 80$

2000 11 (b) (i) $2x + y \le 34,\ 5x + 3y \le 90$
(ii) A = 12, B = 10
(iii) £1320

1999 11 (b) (i) $x + y \le 10,\ x + 3y \le 18$
(ii) $x = 6,\ y = 4$

1998 11 (b) (i) $3x + y \le 60,\ x + 2y \le 40$
(ii) 16 of A and 12 of B
(iii) A: £90; B: £45

1997 11 (b) (i) $x + y \le 500,\ 4x + 3y \le 1800$
(ii) $x = 450,\ y = 0$

1996 11 (b) (i) $3x + y \le 48,\ 5x + 3y \le 120$
(ii) $x = 0,\ y = 40$; £5600
(iii) $x = 0,\ y = 40$