The Line (Q 2, Paper 2)

Lesson No. 8: Area of a Triangle

2007

2 (c) $a(-4, 3)$, $b(6, -1)$ and $c(2, 7)$ are three points.

(i) Find the area of triangle abc.

(ii) $abcd$ is a parallelogram in which $[ac]$ is a diagonal.
Find the co-ordinates of the point d.

Solution

2 (c) (i) The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{A = \frac{1}{2}|x_1y_2 - x_2y_1|}$ **6**

Steps
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

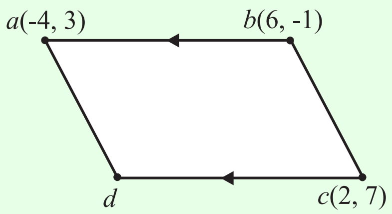
$a(-4, 3) \rightarrow (0, 0)$
$b(6, -1) \rightarrow (10, -4)$
$c(2, 7) \rightarrow (6, 4)$

Translate a to (0, 0) by adding 4 to the x part and taking 3 away from the y part. Do the same to the other two points.

$$a(10, -4) \quad b(6, 4)$$
$$\downarrow \; \downarrow \qquad \downarrow \; \downarrow$$
$$x_1 \; y_1 \qquad x_2 \; y_2$$

$$A = \tfrac{1}{2}|(10)(4) - (-4)(6)|$$
$$\Rightarrow A = \tfrac{1}{2}|40 + 24|$$
$$\therefore A = \tfrac{1}{2}|64| = 32 \text{ square units}$$

2 (c) (ii) Sketch the points abc. The fourth point, d, of the parallelogram $abcd$ is obtained by finding the image of c under a translation from b to a.



$b(6, -1) \rightarrow a(-4, 3)$
$c(2, 7) \rightarrow d(-8, 11)$

2006

2 (a) $a(-2, 6)$ and $b(4, 3)$ are two points.

(i) Plot a and b on a co-ordinate diagram.

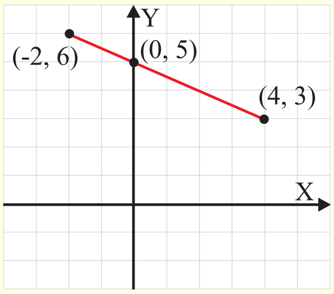
(ii) From your diagram, write down the co-ordinates of the point at which the line ab cuts the y-axis.

(iii) Find the slope of ab.

(iv) Calculate the area of the triangle abc, where the co-ordinates of c are $(1, -3)$.

SOLUTION

2 (a) (i)



2 (a) (ii)

You can see the line cuts the y-axis at (0, 5).

2 (a) (iii)

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}} \;\ldots\ldots\; 3$$

REMEMBER IT AS:

Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\boxed{\begin{array}{cc} a(-2, 6) & b(4, 3) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ \ y_1 & x_2\ \ y_2 \end{array}}$$

$$m = \frac{3-6}{4-(-2)} = \frac{-3}{4+2} = \frac{-3}{6} = -\frac{1}{2}$$

2 (a) (iv)

The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$$\boxed{A = \tfrac{1}{2}|x_1y_2 - x_2y_1|} \;\ldots\ldots\; 6$$

STEPS

1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

$a(-2, 6) \rightarrow (-3, 9)$

$b(4, 3) \rightarrow (3, 6)$

$c(1, -3) \rightarrow (0, 0)$

Translate c to (0, 0) by taking 1 away from the x part and adding 3 to the y part. Do the same to the other two points.

$$\boxed{\begin{array}{cc} a(-3, 9) & b(3, 6) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ \ y_1 & x_2\ \ y_2 \end{array}}$$

$$A = \tfrac{1}{2}|(-3)(6) - (9)(3)|$$

$$\Rightarrow A = \tfrac{1}{2}|-18 - 27|$$

$$\therefore A = \tfrac{1}{2}|-45| = \tfrac{45}{2} \text{ square units}$$

2004

2 (b) $a(-1, -2)$, $b(3, 1)$, $c(0, 4)$ are three points.

(i) Find the length of $[ab]$.

(ii) Calculate the area of the triangle abc.

(iii) The line L is parallel to ab and passes through the point c.
Find the equation of L.

(iv) Show that the point $d(-4, 1)$ is on L.

(v) Investigate whether $abcd$ is a parallelogram.

SOLUTION

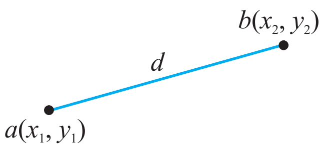
2 (b) (i)

$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$ **1**

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$



$$\begin{array}{cccc} a(-1, & -2) & b(3, & 1) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$|ab| = \sqrt{(3-(-1))^2 + (1-(-2))^2}$$
$$\Rightarrow |ab| = \sqrt{(3+1)^2 + (1+2)^2}$$
$$\Rightarrow |ab| = \sqrt{4^2 + 3^2} = \sqrt{16+9}$$
$$\therefore |ab| = \sqrt{25} = 5$$

2 (b) (ii)

The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{A = \frac{1}{2}|x_1 y_2 - x_2 y_1|}$ **6**

STEPS
1. Translate one point to $(0, 0)$.
2. Do the same translation to the other two points.
3. Apply the formula.

$a(-1, -2) \rightarrow (0, 0)$
$b(3, 1) \rightarrow (4, 3)$
$c(0, 4) \rightarrow (1, 6)$

Translate a to $(0, 0)$ by adding 1 to the x part and adding 2 to the y part. Do the same to the other two points.

$$\begin{array}{cccc} (4, & 3) & (1, & 6) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$A = \tfrac{1}{2}|(4)(6) - (3)(1)|$$
$$\Rightarrow A = \tfrac{1}{2}|24 - 3| = \tfrac{1}{2}|21|$$
$$\therefore A = \tfrac{21}{2} \text{ square units}$$

CONT.....

2 (b) (iii)

Firstly, find the slope of *ab*.

$$\boxed{m=\frac{y_2-y_1}{x_2-x_1}} \quad \ldots\ldots\ \textbf{3}$$

REMEMBER IT AS:

Slope $m=\dfrac{\text{Difference in } y'\text{s}}{\text{Difference in } x'\text{s}}$

$$\begin{array}{cccc} a(-1, & -2) & b(3, & 1) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$m=\frac{1-(-2)}{3-(-1)}=\frac{1+2}{3+1}=\frac{3}{4}$$

Parallel lines have the same slope.

The slope of L is also $\frac{3}{4}$.

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y-y_1=m(x-x_1)}$ **4**

Equation of L: Point $(x_1, y_1)=(0, 4)$, slope $m=\frac{3}{4}$.

$y-y_1=m(x-x_1)$

$\Rightarrow y-4=\frac{3}{4}(x-0)$

$\Rightarrow 4(y-4)=3x$

$\Rightarrow 4y-16=3x$

$\therefore 3x-4y+16=0$

2 (b) (iv)

IS A POINT ON A LINE?

To show a point is on a line, put the point into the equation.

If you put the point *d* into *L*, it will satisfy the equation.

$d(-4, 1)\in L?\ 3(-4)-4(1)+16=-12-4+16$

$=0\Rightarrow d(-4, 1)\in L$

2 (b) (v)

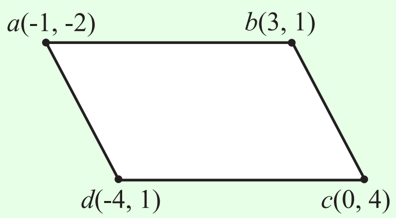
To prove *abcd* is a parallelogram: Find the slopes of each side and show opposite sides have the same slopes.

Slope of *ab*: $m=\frac{3}{4}$

Slope of *dc*: $m=\dfrac{1-4}{-4-0}=\dfrac{-3}{-4}=\dfrac{3}{4}$

Slope of *ad*: $m=\dfrac{1-(-2)}{-4-(-1)}=\dfrac{1+2}{-4+1}=\dfrac{3}{-3}=-1$

Slope of *bc*: $m=\dfrac{1-4}{3-0}=\dfrac{-3}{3}=-1$



Opposite sides have the same slope. Therefore, *abcd* is a parallelogram.

2002

2 (b) The line L has equation $4x-5y=-40$.

$a(0, 8)$ and $b(-10, 0)$ are two points.

(i) Verify that a and b lie on L.

(ii) What is the slope of L?

(iii) The line K is perpendicular to L and it contains b. Find the equation of K.

(iv) K intersects the y-axis at the point c. Find the co-ordinates of c.

(v) d is another point such that $abcd$ is a rectangle. Calculate the area of $abcd$.

(vi) Find the co-ordinates of d.

SOLUTION

2 (b) (i)

IS A POINT ON A LINE?
To show a point is on a line, put the point into the equation.

$a(0, 8) \in L$?

$4x-5y=4(0)-5(8)$

$=0-40$

$=-40 \Rightarrow a(0, 8) \in L$

$b(-10, 0) \in L$?

$4x-5y=4(-10)-5(0)$

$=-40+0$

$=-40 \Rightarrow b(-10, 0) \in L$

2 (b) (ii)

GENERAL FORM OF A STRAIGHT LINE
Every straight line can be written in the form: $ax + by + c = 0$.
You can read off the slope of a straight line from its equation.

Slope: $\boxed{m=-\left(\frac{a}{b}\right)}$ 5

REMEMBER IT AS: Slope $m=-\left(\frac{\text{Number in front of } x}{\text{Number in front of } y}\right)$

Slope of L: $m=\frac{4}{5}$

2 (b) (iii)

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y-y_1=m(x-x_1)}$ 4

CONT.....

Equation of K: Slope $m = -\frac{5}{4}$, point $(x_1, y_1) = (-10, 0)$

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 0 = -\tfrac{5}{4}(x - (-10))$$
$$\Rightarrow y = -\tfrac{5}{4}(x + 10)$$
$$\Rightarrow 4y = -5(x + 10)$$
$$\Rightarrow 4y = -5x - 50$$
$$\therefore 5x + 4y + 50 = 0$$

2 (b) (iv)

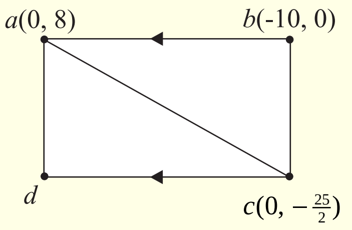
To find the x-intercept: Put $y = 0$.
To find the y-intercept: Put $x = 0$.

Put $x = 0$:

$$\therefore 5(0) + 4y + 50 = 0$$
$$\Rightarrow 4y = -50$$
$$\therefore y = -\tfrac{50}{4} = -\tfrac{25}{2}$$
$\Rightarrow c(0, -\frac{25}{2})$ is the y intercept.

2 (b) (v)

The rectangle *abcd* is bisected by the diagonal *ac*. Find the area of triangle *abc* and double it to find the area of *abcd*.



The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{A = \frac{1}{2}|x_1y_2 - x_2y_1|}$ 6

STEPS
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

$$a(0, 8) \rightarrow (0, 0)$$
$$b(-10, 0) \rightarrow (-10, -8)$$
$$c(0, -\tfrac{25}{2}) \rightarrow (0, -\tfrac{41}{2})$$

Translate a to (0, 0) by leaving the x part unchanged and taking 8 away from the y part. Do the same to the other two points.

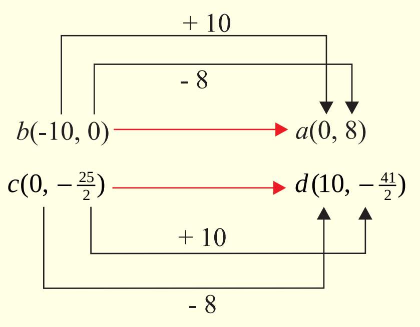
Area of triangle *abc*: $A = \frac{1}{2}|(-10)(-\frac{41}{2}) - (-8)(0)|$

$$\Rightarrow A = \tfrac{1}{2}|205 + 0| = \tfrac{205}{2}$$

Area of rectangle *abcd*: A = 205 square units

2 (b) (vi)

To find d, do a translation.



2001

2 (b) $a(4, 2)$, $b(-2, 0)$ and $c(0, 4)$ are three points.

(i) Prove that $ac \perp bc$.

(ii) Prove that $|ac| = |bc|$.

(iii) Calculate the area of the triangle bac.

(iv) The diagonals of the square $bahg$ intersect at c.
Find the co-ordinates of h and the co-ordinates of g.

(v) Find the equation of the line bc and show that h lies on this line.

SOLUTION

2 (b) (i)

$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$ 3

REMEMBER IT AS:

Slope $m = \frac{\text{Difference in } y'\text{s}}{\text{Difference in } x'\text{s}}$

$$\begin{array}{cc} a(4,\ 2) & c(0,\ 4) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

Slope of ac: $m_1 = \frac{4-2}{0-4} = \frac{2}{-4} = -\frac{1}{2}$

$$\begin{array}{cc} b(-2,\ 0) & c(0,\ 4) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

Slope of bc: $m_2 = \frac{4-0}{0-(-2)} = \frac{4}{2} = 2$

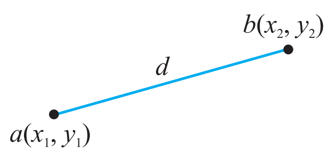
Two lines are perpendicular if the product of their slopes is -1.

$$m_1 \times m_2 = (-\tfrac{1}{2}) \times 2 = -1 \Rightarrow ac \perp bc$$

$\boxed{K \perp L \Rightarrow m_1 \times m_2 = -1}$

2 (b) (ii)

$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$ 1



The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$d = \sqrt{(\text{Difference in } x'\text{s})^2 + (\text{Difference in } y'\text{s})^2}$

$$\begin{array}{cc} a(4,\ 2) & c(0,\ 4) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$$|ac| = \sqrt{(0-4)^2 + (4-2)^2}$$
$$\Rightarrow |ac| = \sqrt{(-4)^2 + (2)^2} = \sqrt{16+4}$$
$$\therefore |ac| = \sqrt{20}$$

$$\begin{array}{cc} b(-2,\ 0) & c(0,\ 4) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$$|bc| = \sqrt{(0-(-2))^2 + (4-0)^2}$$
$$\Rightarrow |bc| = \sqrt{(2)^2 + (4)^2} = \sqrt{4+16}$$
$$\therefore |bc| = \sqrt{20}$$

$$\therefore |ac| = |bc|$$

CONT.....

2 (b) (iii) The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{A = \frac{1}{2}|x_1 y_2 - x_2 y_1|}$ **6**

Steps
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

$a(4, 2) \rightarrow (6, 2)$
$b(-2, 0) \rightarrow (0, 0)$
$c(0, 4) \rightarrow (2, 4)$

Translate b to (0, 0) by adding 2 to the x part and adding 0 to the y part. Do the same to the other two points.

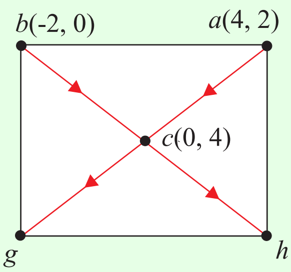
(6, 2)	(2, 4)
$\downarrow\ \downarrow$	$\downarrow\ \downarrow$
$x_1\ y_1$	$x_2\ y_2$

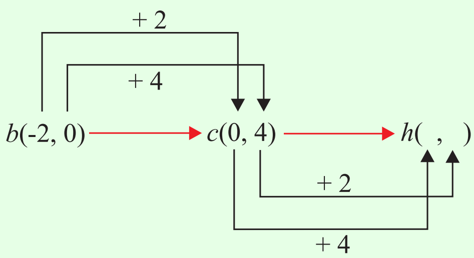
$A = \frac{1}{2}|(6)(4) - (2)(2)|$

$\Rightarrow A = \frac{1}{2}|24 - 4| = \frac{1}{2}|20|$

$\therefore A = 10$ square units

2 (b) (iv)
The diagonals of a square bisect each other. To find out the co-ordinates of g and h send points b and a through c by a central symmetry.

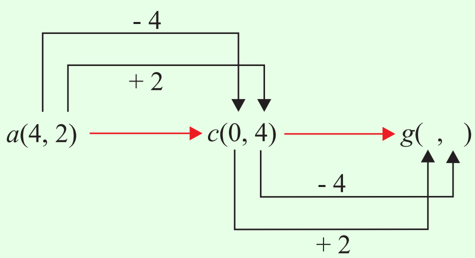




To go from b to c, you add 2 to the x-coordinate and 4 to the y-coordinate. Therefore, to go from c to h, you do exactly the same.

$\therefore b(-2, 0) \rightarrow c(0, 4) \rightarrow h(2, 8)$

Note: c is the midpoint of $[bh]$.



To go from a to c, you take away 4 from the x-coordinate and add 2 to the y-coordinate. Therefore, to go from c to g, you do exactly the same.

$\therefore a(4, 2) \rightarrow c(0, 4) \rightarrow g(-4, 6)$

Ans: $g(-4, 6)$, $h(2, 8)$

2 (b) (v)

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ **4**

Equation of bc: Point $c(0, 4)$, slope $m = 2$ [found in **2 (b) (i)**]

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 4 = 2(x - 0)$

$\Rightarrow y - 4 = 2x$

$\therefore 2x - y + 4 = 0$

Cont.....

Is a point on a line?
To show a point is on a line, put the point into the equation.

$h(2, 8) \in bc$?

$2(2)-(8)+4$

$=4-8+4$

$=0 \Rightarrow h(2, 8) \in bc$

2000

2 (c) (i) The line L has equation $3x - 4y + 20 = 0$.
K is the line through $p(0, 5)$ which is perpendicular to L.
Find the equation of K.

(ii) L cuts the x-axis at the point t.
K cuts the x-axis at the point r.
Calculate the area of the triangle ptr. Give your answer as a fraction.

Solution

2 (c) (i)

General form of a straight line
Every straight line can be written in the form: $ax + by + c = 0$.
You can read off the slope of a straight line from its equation.

Slope: $\boxed{m=-\left(\frac{a}{b}\right)}$ 5

Remember it as: Slope $m=-\left(\frac{\text{Number in front of } x}{\text{Number in front of } y}\right)$

$L: 3x-4y+20=0 \Rightarrow m=\frac{3}{4}$

Finding the perpendicular slope: Invert the slope and change its sign.

K is perpendicular to L.

Equation of K: Point $p(0, 5)$, slope $m=-\frac{4}{3}$.

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y-y_1=m(x-x_1)}$ 4

$y-y_1=m(x-x_1)$

$\Rightarrow y-5=-\frac{4}{3}(x-0)$

$\Rightarrow 3(y-5)=-4x$

$\Rightarrow 3y-15=-4x$

$\therefore 4x+3y-15=0$

Cont.....

2 (c) (ii)

Finding where L cuts the x-axis.

$y=0$: $3x-4(0)+20=0$

$\Rightarrow 3x=-20$

$\Rightarrow x=-\frac{20}{3} \Rightarrow (-\frac{20}{3},\ 0)$ is the x-intercept.

To find the x-intercept: Put $y = 0$.
To find the y-intercept: Put $x = 0$.

Finding where K cuts the x-axis.

$y=0$: $4x+3(0)-15=0$

$\Rightarrow 4x=15$

$\therefore x=\frac{15}{4} \Rightarrow (\frac{15}{4},\ 0)$ is the x-intercept.

The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{A=\frac{1}{2}|x_1y_2-x_2y_1|}$ **6**

Steps
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

$p(0,\ 5) \rightarrow (0,\ 0)$

$t(-\frac{20}{3},\ 0) \rightarrow (-\frac{20}{3},\ -5)$

$r(\frac{15}{4},\ 0) \rightarrow (\frac{15}{4},\ -5)$

Translate p to (0, 0) by adding 0 to the x part and taking 5 away from the y part. Do the same to the other two points.

$$\begin{array}{cccc} (-\frac{20}{3}, & -5) & (\frac{15}{4}, & -5) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$A=\frac{1}{2}\left|(-\frac{20}{3})(-5)-(-5)(\frac{15}{4})\right|$

$\Rightarrow A=\frac{1}{2}\left|\frac{100}{3}+\frac{75}{4}\right|$

$\therefore A=\frac{625}{24}$ square units

1997

2 (b) L is the line $x - 2y + 2 = 0$.

M is the line $3x + y - 8 = 0$.

Find the co-ordinates of p, the point of intersection of L and M.

L and M cut the x-axis at q and r, respectively.

Find the area of triangle pqr.

SOLUTION

$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$ 3

REMEMBER IT AS:

Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cc} a(0,\ 4) & b(3,\ 0) \\ \downarrow\downarrow & \downarrow\downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

Slope of ab: $m = \dfrac{0-4}{3-0} = \dfrac{-4}{3} = -\dfrac{4}{3}$

Equation of K: Point $a(4, 0) = (x_1, y_1)$, slope $m = -\frac{4}{3}$

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ 4

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 4 = -\frac{4}{3}(x - 0)$

$\Rightarrow 3(y - 4) = -4x$

$\Rightarrow 3y - 12 = -4x$

$\Rightarrow 4x + 3y - 12 = 0$

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

Equation of N: Point $(x_1, y_1) = (0, 0)$, slope $m = \frac{3}{4}$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = \frac{3}{4}(x - 0)$

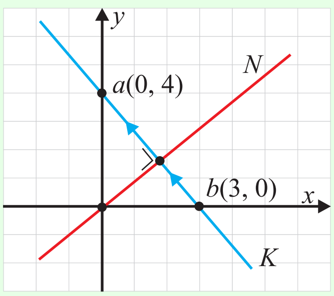
$\Rightarrow 4y = 3x$

$\therefore 3x - 4y = 0$

To answer the last part plot the points and lines to give you a better idea of how to proceed.

AXIAL SYMMETRY: This is the movement of a point perpendicular to a line and out the same distance at right angles to the line.

If b is the image of a under an axial symmetry in N, then the point of intersection of K and N must be the same as the midpoint of $[ab]$.



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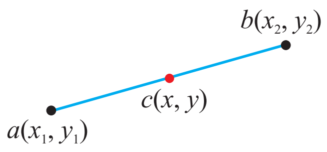
The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\text{Midpoint } = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \ldots\ldots\ \mathbf{2}$$



Remember the midpoint formula as: $\text{Midpoint } = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2} \right)$

$$\begin{array}{cc} a(0,\ 4) & b(3,\ 0) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$$\text{Midpoint } = \left(\frac{0+3}{2}, \frac{4+0}{2} \right) = \left(\frac{3}{2}, \frac{4}{2} \right) = \left(\tfrac{3}{2},\ 2\right)$$

Find the point of intersection of K and N

Intersecting Lines
To find out where two lines intersect, solve their equations **simultaneously**.

$$\begin{aligned} 4x + 3y - 12 &= 0 \ldots(\mathbf{1})(\times 4) \\ 3x - 4y \qquad &= 0 \ldots(\mathbf{2})(\times 3) \end{aligned}$$

$$\begin{aligned} 16x + 12y - 48 &= 0 \\ 9x - 12y \qquad &= 0 \\ \hline 25x \qquad - 48 = 0 \Rightarrow 25x &= -48 \Rightarrow x = -\tfrac{48}{25} \end{aligned}$$

You can see the x value of the point of intersection does not match the the x value of the midpoint. Therefore, b is not an image of a under an axial symmetry in N.

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2 (a) The line L contains the points $p(3, -1)$ and $q(0, 2)$.

(i) Find the slope of L.

(ii) Find the equation of L.

(iii) L intersects the x-axis at the point r. Find the coordinates of r.

(iv) Calculate the ratio

$$\frac{\text{area of triangle } rpo}{\text{area of triangle } pqo}$$

where o is the origin.

SOLUTION

2 (a) (i)

$\boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$ 3

REMEMBER IT AS:

Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$p(3, -1)$ $q(0, 2)$

$\downarrow \ \downarrow \quad \downarrow \ \downarrow$

$x_1 \ y_1 \quad x_2 \ y_2$

Slope of L: $m = \dfrac{2-(-1)}{0-3} = \dfrac{2+1}{0-3} = \dfrac{3}{-3} = -1$

2 (a) (ii)

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ 4

Equation of L: point $q(0, 2) = (x_1, x_2)$, slope $m = -1$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = -1(x - 0)$

$\Rightarrow y - 2 = -x$

$\therefore x + y - 2 = 0$

2 (a) (iii)

To find the x-intercept: Put $y = 0$.
To find the y-intercept: Put $x = 0$.

To find the x-intercept of L, put $y = 0$.

$y = 0: x + (0) - 2 = 0 \Rightarrow x = 2 \Rightarrow r(2, 0)$ is the x-intercept.

CONT.....

2 (a) (iv)

The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{y = 0 : x + (0) -}$ **6**

STEPS
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

Area of triangle *rpo*: One of the points is already (0, 0) so all you have to do is apply the formula to the other 2 points.

$$\begin{array}{cc} r(2,\ 0) & p(3,\ -1) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ \ y_1 & x_2\ \ y_2 \end{array}$$

$A_1 = \frac{1}{2}|(2)(-1) - (0)(3)|$

$\Rightarrow A_1 = \frac{1}{2}|-2 - 0| = \frac{1}{2}|-2| = \frac{1}{2}(2)$

$\therefore A_1 = 1$ square unit

Area of triangle *pqo*:

$$\begin{array}{cc} p(3,\ -1) & q(0,\ 2) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ \ y_1 & x_2\ \ y_2 \end{array}$$

$A_2 = \frac{1}{2}|(3)(2) - (-1)(0)|$

$\Rightarrow A_2 = \frac{1}{2}|6 + 0| = \frac{1}{2}|6| = \frac{1}{2}(6)$

$\therefore A_2 = 3$ square units

$$\frac{A_1}{A_2} = \frac{1}{3} \Rightarrow A_1 : A_2 = 1:3$$