THE LINE (Q 2, PAPER 2)

LESSON NO. 7: TRANSLATIONS & SYMMETRIES

2006

2 (b) L is the line $3x + 2y + c = 0$.

(i) $(3, -1)$ is a point on L. Find the value of c.

(ii) The line K is parallel to L and passes through the point $(-2, 5)$. Find the equation of K.

(iii) The lines L and K, together with the line $x = 3$ and the y-axis, form a parallelogram. Find the co-ordinates of the vertices of the parallelogram.

SOLUTION

2 (b) (i)

IS A POINT ON A LINE?
To show a point is on a line, put the point into the equation.

You are told that $(3, -1)$ is on L. Therefore, you can substitute it in to find c. Replace x by 3 and y by -1.

$(3, -1) \in L \Rightarrow 3(3) + 2(-1) + c = 0$

$\Rightarrow 9 - 2 + c = 0$

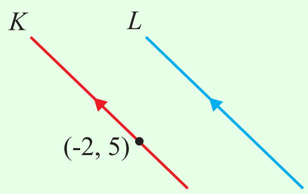
$\Rightarrow 7 + c = 0$

$\therefore c = -7$

2 (b) (ii)

Parallel lines have the same slope.

$\boxed{K \parallel L \Rightarrow m_1 = m_2}$



Equation of L: $3x + 2y - 7 = 0$

Slope: $\boxed{m = -\left(\frac{a}{b}\right)}$ .... 5 **REMEMBER IT AS:** Slope $m = -\left(\frac{\text{Number in front of } x}{\text{Number in front of } y}\right)$

Slope of L: $m = -\frac{3}{2}$

Therefore, slope of K: $m = -\frac{3}{2}$

Equation of K: slope $m = -\frac{3}{2}$, point $(x_1, y_1) = (-2, 5)$.

$(x_1, y_1) = (-2, 5)$.

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 5 = -\frac{3}{2}(x - (-2))$

$\Rightarrow 2(y - 5) = -3(x + 2)$

$\Rightarrow 2y - 10 = -3x - 6$

$\therefore 3x + 2y - 4 = 0$

CONT....

2 (b) (ii)

The y-axis has an equation $x = 0$. This is parallel to the line $x = 3$. You need to find out where the lines L and K intersect these lines.

Points of intersection of L: $3x + 2y - 7 = 0$ with:

$x = 0$:

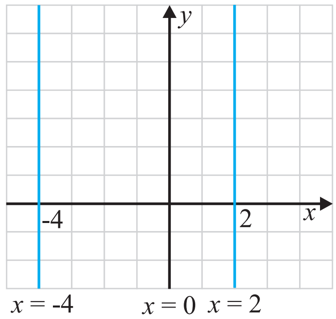
$3(0) + 2y - 7 = 0 \Rightarrow 2y = 7$

$\Rightarrow y = \frac{7}{2} \Rightarrow (0, \frac{7}{2})$ is a point of intersection.

$x = 3$:

$3(3) + 2y - 7 = 0 \Rightarrow 9 + 2y = 7$

$\Rightarrow 2y = -2 \Rightarrow y = -1 \Rightarrow (0, -1)$ is a point of intersection.



Vertical lines have equations where x = constant. In particular, the y-axis has the equation $x = 0$.

Points of intersection of K: $3x + 2y - 4 = 0$ with:

$x = 0$:

$3(0) + 2y - 4 = 0 \Rightarrow 0 + 2y = 4$

$\Rightarrow y = 2 \Rightarrow (0, 2)$ is a point of intersection.

$x = 3$:

$3(3) + 2y - 4 = 0 \Rightarrow 9 + 2y = 4$

$\Rightarrow 2y = -5 \Rightarrow y = -\frac{5}{2} \Rightarrow (3, -\frac{5}{2})$ is a point of intersection.

The co-ordinates of the vertices of the parallelogram are: $(0, 2), (3, -\frac{5}{2}), (0, \frac{7}{2}), (3, -1)$

1998

2 (b) $a(2, -1)$, $b(-2, 3)$, $c(-1, -1)$ and $d(4, -6)$ are points.

(i) Show that ab is parallel to cd.

(ii) Investigate if $abcd$ is a parallelogram.
Give a reason for your answer.

SOLUTION

2 (b) (i)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{.......} \ 3$$

REMEMBER IT AS:

Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cccc} a(2, & -1) & b(-2, & 3) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of ab: $m_1 = \dfrac{3-(-1)}{-2-2} = \dfrac{3+1}{-4} = \dfrac{4}{-4} = -1$

$$\begin{array}{cccc} c(-1, & -1) & d(4, & -6) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of cd: $m_2 = \dfrac{-6-(-1)}{4-(-1)} = \dfrac{-6+1}{4+1} = \dfrac{-5}{5} = -1$

Parallel lines have the same slope.

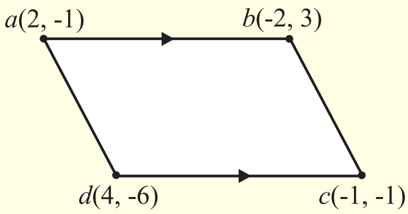
$m_1 = m_2 \Rightarrow ab \parallel cd$

CONT....

2 (b) (ii)

To prove *abcd* is a parallelogram: Find the slopes of each side and show opposite sides have the same slopes.

You have shown in part **(i)** that $ab \parallel cd$.
To show *abcd* is a parallelogram you need to also show that $ad \parallel bc$.



$$\begin{array}{cccc} a(2, -1) & & d(4, -6) & \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of *ad*: $m_3 = \dfrac{-6-(-1)}{4-2} = \dfrac{-6+1}{4-2} = \dfrac{-5}{2} = -\dfrac{5}{2}$

$$\begin{array}{cccc} b(-2, 3) & & c(-1, -1) & \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of *bc*: $m_4 = \dfrac{-1-3}{-1-(-2)} = \dfrac{-1-3}{-1+2} = \dfrac{-4}{1} = -4$

ad is not parallel to *bc*. Therefore, *abcd* is not a parallelogram.

1997

2 (c) *K* is the line which contains the points *a*(0, 4) and *b*(3, 0).
Find the equation of *K*.
N is the line which is perpendicular to *K* and which contains the origin.
Find the equation of *N*.
Investigate if *b* is the image of *a* under the axial symmetry in *N*.

SOLUTION

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ 3

REMEMBER IT AS:

Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cccc} a(0, 4) & & b(3, 0) & \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of *ab*: $m = \dfrac{0-4}{3-0} = \dfrac{-4}{3} = -\dfrac{4}{3}$

Equation of *K*: Point *a*(4, 0) = (x_1, y_1), slope $m = -\frac{4}{3}$

The equation of a line is a formula satisfied by every point (*x*, *y*) on the line.

Equation of a line: $y - y_1 = m(x - x_1)$ 4

$y - y_1 = m(x - x_1)$
$\Rightarrow y - 4 = -\frac{4}{3}(x - 0)$
$\Rightarrow 3(y-4) = -4x$
$\Rightarrow 3y - 12 = -4x$
$\Rightarrow 4x + 3y - 12 = 0$

CONT....

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

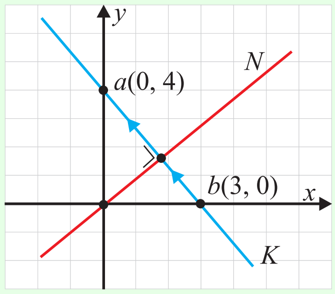
Equation of N: Point $(x_1, y_1) = (0, 0)$, slope $m = \frac{3}{4}$

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 0 = \tfrac{3}{4}(x - 0)$$
$$\Rightarrow 4y = 3x$$
$$\therefore 3x - 4y = 0$$

To answer the last part plot the points and lines to give you a better idea of how to proceed.

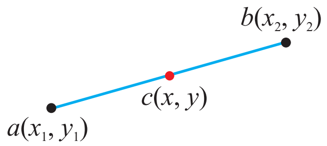
AXIAL SYMMETRY: This is the movement of a point perpendicular to a line and out the same distance at right angles to the line.

If b is the image of a under an axial symmetry in N, then the point of intersection of K and N must be the same as the midpoint of $[ab]$.



The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \text{.......} \quad \textbf{2}$$



REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint} = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2} \right)$

$$\begin{array}{cc} a(0,\ 4) & b(3,\ 0) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$$\text{Midpoint} = \left(\frac{0+3}{2}, \frac{4+0}{2} \right) = \left(\frac{3}{2}, \frac{4}{2} \right) = (\tfrac{3}{2}, 2)$$

Find the point of intersection of K and N

INTERSECTING LINES
To find out where two lines intersect, solve their equations **simultaneously**.

$$\begin{aligned} 4x + 3y - 12 &= 0 \ldots(\mathbf{1})(\times 4) \\ 3x - 4y \qquad &= 0 \ldots(\mathbf{2})(\times 3) \end{aligned}$$

$$\rightarrow$$

$$\begin{aligned} 16x + 12y - 48 &= 0 \\ 9x - 12y \qquad &= 0 \\ \hline 25x \qquad - 48 &= 0 \Rightarrow 25x = -48 \Rightarrow x = -\tfrac{48}{25} \end{aligned}$$

You can see the x value of the point of intersection does not match the the x value of the midpoint. Therefore, b is not an image of a under an axial symmetry in N.