LESSON NO. 5: EQUATION OF A LINE I

2007

2 (b) The line L intersects the x-axis at $(-4, 0)$ and the y-axis at $(0, 6)$.

(i) Find the slope of L.

(ii) Find the equation of L.

The line K passes through $(0, 0)$ and is perpendicular to L.

(iii) Show the lines L and K on a co-ordinate diagram.

(iv) Find the equation of K.

SOLUTION

2 (b) (i)

$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$ 3

REMEMBER IT AS:

Slope $m = \frac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$a(-4, 0)$ $b(0, 6)$

$\begin{matrix} \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$

$$m = \frac{6-0}{0-(-4)} = \frac{6}{4} = \frac{3}{2}$$

2 (b) (ii)

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ 4

Slope $m = \frac{3}{2}$, given point $(x_1, y_1) = (-4, 0)$.

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (0) = \frac{3}{2}(x - (-4))$

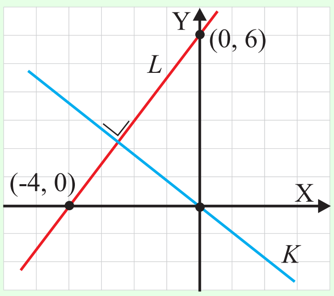
$\Rightarrow y = \frac{3}{2}(x + 4)$

$\Rightarrow 2y = 3(x + 4)$

$\Rightarrow 2y = 3x + 12$

$\therefore 3x - 2y + 12 = 0$

2 (b) (iii)



2 (b) (iv)

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

Equation of K: The slope is perpendicular to L. $\therefore m = -\frac{2}{3}$. Point $(x_1, y_1) = (0, 0)$.

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = -\frac{2}{3}(x - 0)$

$\Rightarrow 3y = -2x$

$\therefore 2x + 3y = 0$

2001

2 (a) The point $(t, 2t)$ lies on the line $3x + 2y + 7 = 0$.
Find the value of t.

SOLUTION

IS A POINT ON A LINE?
To show a point is on a line, put the point into the equation.

Replace x by t and y by $2t$.

$(t,\ 2t) \in 3x + 2y + 7 = 0$

$\Rightarrow 3(t) + 2(2t) + 7 = 0$

$\Rightarrow 3t + 4t + 7 = 0$

$\Rightarrow 7t = -7$

$\therefore t = -1$

1999

2 (a) The point $(k, 1)$ lies on the line $4x - 3y + 15 = 0$.
Find the value of k.

(b) $p(4, 3)$, $q(-1, 0)$ and $r(10, 3)$ are three points.

(i) Find the slope of pq.

(ii) Find the equation of the line through r which is parallel to pq.

(iii) Find the equation of the line which is perpendicular to pq and which contains the origin.

SOLUTION

2 (a)

IS A POINT ON A LINE?
To show a point is on a line, put the point into the equation.

$(k, 1) \in 4x - 3y + 15 = 0$

$\Rightarrow 4(k) - 3(1) + 15 = 0$

$\Rightarrow 4k - 3 + 15 = 0$

$\Rightarrow 4k + 12 = 0$

$\Rightarrow 4k = -12$

$\therefore k = -3$

CONT....

2 (b) (i)

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ 3

Remember it as:

Slope $m = \dfrac{\text{Difference in } y'\text{s}}{\text{Difference in } x'\text{s}}$

$p(4, 3)$ $q(-1, 0)$
$\downarrow\downarrow$ $\downarrow\downarrow$
$x_1\ y_1$ $x_2\ y_2$

Slope of pq: $m = \dfrac{0-3}{-1-4} = \dfrac{-3}{-5} = \dfrac{3}{5}$

2 (b) (ii)

Equation of line: Point $r(10, 3)$, slope $m = \frac{3}{5}$.

Parallel lines have the same slope.

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = \frac{3}{5}(x - 10)$

$\Rightarrow 5(y - 3) = 3(x - 10)$

$\Rightarrow 5y - 15 = 3x - 30$

$\therefore 3x - 5y - 15 = 0$

2 (b) (iii)

Finding the perpendicular slope: Invert the slope and change its sign.

Equation of line: Point $(0, 0)$, slope $m = -\frac{5}{3}$.

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = -\frac{5}{3}(x - 0)$

$\Rightarrow 3y = -5x$

$\therefore 5x + 3y = 0$

1998

2 (a) The point $(-3, 4)$ is on the line whose equation is $5x + y + k = 0$. Find the value of k.

Solution

Is a point on a line?
To show a point is on a line, put the point into the equation.

$(-3, 4) \in 5x + y + k = 0$

$\Rightarrow 5(-3) + (4) + k = 0$

$\Rightarrow -15 + 4 + k = 0$

$\Rightarrow -11 + k = 0$

$\therefore k = 11$

1996

2 (b) The equation of the line M is $y-4x-c=0$.
M contains the point $p(1, 6)$.
(i) Find the value of c.

(ii) The origin is the midpoint of $[pq]$.
Find the equation of the line K if K is parallel to M and K contains the point q.

(iii) Find the equation of the line L if L is perpendicular to M and L contains the point q.

SOLUTION

2 (b) (i)

> **IS A POINT ON A LINE?**
> To show a point is on a line, put the point into the equation.

$p(1, 6) \in y-4x-c=0$
$\Rightarrow (6)-4(1)-c=0$
$\Rightarrow 6-4-c=0$
$\Rightarrow 2-c=0$
$\therefore c=2$

2 (b) (ii)

To find the equation of K you need a point on K and the slope of K.

> **GENERAL FORM OF A STRAIGHT LINE**
> Every straight line can be written in the form: $ax+by+c=0$.
> You can read off the slope of a straight line from its equation.
>
> Slope: $\boxed{m=-\left(\frac{a}{b}\right)}$ 5
>
> **REMEMBER IT AS:** Slope $m=-\left(\dfrac{\text{Number in front of } x}{\text{Number in front of } y}\right)$

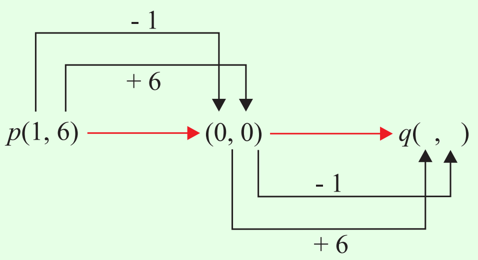
$M: y-4x-2=0 \Rightarrow 4x-y+2=0$

Slope of M: $m=-\frac{4}{-1}=4$

> Parallel lines have the same slope.

Slope of K: $m=4$

The image of $p(1, 6)$ through the origin $(0, 0)$ by a central symmetry is the point q.



$\therefore p(1, 6) \rightarrow (0, 0) \rightarrow q(-1, -6)$

CONT....

Equation of K: Point $(x_1, y_1) = q(-1, -6)$, slope $m = 4$.

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (-6) = 4(x - (-1))$

$\Rightarrow y + 6 = 4(x + 1)$

$\Rightarrow y + 6 = 4x + 4$

$\therefore 4x - y - 2 = 0$

2 (b) (iii)

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

Equation of L: Point $(x_1, y_1) = q(-1, -6)$, slope $m = -\frac{1}{4}$.

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (-6) = -\frac{1}{4}(x - (-1))$

$\Rightarrow 4(y + 6) = -1(x + 1)$

$\Rightarrow 4y + 24 = -x - 1$

$\therefore x + 4y + 25 = 0$