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THE LINE (Q 2, PAPER 2)

LESSON NO. 2: DISTANCE FORMULA

2005

2 (a) Find the distance between the two points (3, 4) and (15, 9).

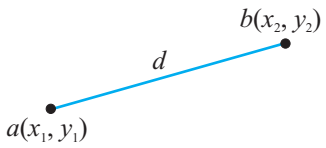
SOLUTION

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots \textbf{1}$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$a(3, 4)$ $b(15, 9)$

$x_1\ y_1$ $x_2\ y_2$

$$|ab| = \sqrt{(15-3)^2 + (9-4)^2}$$

$$\Rightarrow |ab| = \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$

$$\therefore \Rightarrow |ab| = \sqrt{169} = 13$$

2003

2 (a) Find the distance between the two points (3, 2) and (8, 14).

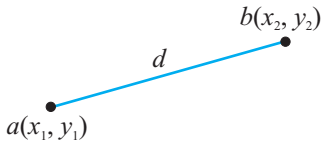
SOLUTION

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots \textbf{1}$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$a(3, 2)$ $b(8, 14)$

$x_1\ y_1$ $x_2\ y_2$

$$|ab| = \sqrt{(8-3)^2 + (14-2)^2}$$

$$\Rightarrow |ab| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144}$$

$$\therefore \Rightarrow |ab| = \sqrt{169} = 13$$

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2000

2 (b) $a(-2, -1)$, $b(1, 0)$ and $c(-5, 2)$ are three points.

(i) Show that $|ab| = \sqrt{10}$.

(ii) Find $|bc|$.

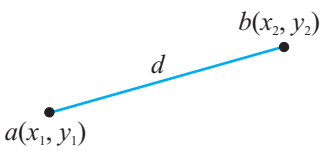
(iii) Hence, find the ratio $|ab| : |bc|$.
Give your answer in the form $m:n$ where m and n are whole numbers.

SOLUTION

2 (b) (i)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots \textbf{1}$$

The distance between a and b is written as $|ab|$.



REMEMBER THE DISTANCE FORMULA AS:

$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$

$a(-2, -1)$ $b(1, 0)$
$\downarrow\ \downarrow$ $\downarrow\downarrow$
$x_1\ y_1$ $x_2\ y_2$

$$|ab| = \sqrt{(1-(-2))^2 + (0-(-1))^2}$$
$$\Rightarrow |ab| = \sqrt{(1+2)^2 + (0+1)^2}$$
$$\Rightarrow |ab| = \sqrt{3^2 + 1^2} = \sqrt{9+1}$$
$$\therefore |ab| = \sqrt{10}$$

2 (b) (ii)

$b(1, 0)$ $c(-5, 2)$
$\downarrow\downarrow$ $\downarrow\downarrow$
$x_1\ y_1$ $x_2\ y_2$

$$|bc| = \sqrt{(-5-1)^2 + (2-0)^2}$$
$$\Rightarrow |bc| = \sqrt{(-6)^2 + (2)^2}$$
$$\Rightarrow |bc| = \sqrt{36+4}$$
$$\therefore |bc| = \sqrt{40} = \sqrt{4\times 10} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$$

2 (b) (iii)

$$|ab| : |bc| = \sqrt{10} : 2\sqrt{10} = 1:2$$

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1999

2 (c) $a(0, 5)$, $b(x, 10)$ and $c(2x, x)$ are three points.
Find $|ab|$ in terms of x.
If $|ab| = |bc|$, calculate the two possible values of x.

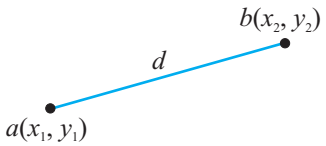
SOLUTION

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots \textbf{1}$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$a(0, 5)$ $b(x, 10)$
$x_1\ y_1$ $x_2\ y_2$

$$|ab| = \sqrt{(10-5)^2 + (x-0)^2}$$
$$\Rightarrow |ab| = \sqrt{5^2 + x^2}$$
$$\therefore |ab| = \sqrt{x^2 + 25}$$

$b(x, 10)$ $c(2x, x)$
$x_1\ y_1$ $x_2\ y_2$

$$|bc| = \sqrt{(2x-x)^2 + (x-10)^2}$$
$$\therefore |bc| = \sqrt{x^2 + (x-10)^2}$$

$$|ab| = |bc|$$
$$\Rightarrow \sqrt{x^2 + 25} = \sqrt{x^2 + (x-10)^2} \quad \text{[Square both sides.]}$$
$$\Rightarrow x^2 + 25 = x^2 + (x-10)^2 \quad \text{[Square out the bracket.]}$$
$$\Rightarrow 25 = x^2 - 20x + 100$$
$$\Rightarrow x^2 - 20x + 75 = 0 \quad \text{[Factorise the quadratic.]}$$
$$\Rightarrow (x-5)(x-15) = 0$$
$$\therefore x = 5, 15$$

1997

2 (a) Find the distance between the two points $(-5, 1)$ and $(7, -4)$.

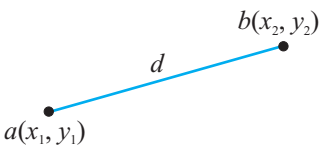
SOLUTION

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots \textbf{1}$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$(-5, 1)$ $(7, -4)$
$x_1\ y_1$ $x_2\ y_2$

$$d = \sqrt{(7-(-5))^2 + (-4-1)^2}$$
$$\Rightarrow d = \sqrt{(7+5)^2 + (-4-1)^2}$$
$$\Rightarrow d = \sqrt{(12)^2 + (-5)^2} = \sqrt{144 + 25}$$
$$\therefore d = \sqrt{169} = 13$$