The Line (Q 2, Paper 2)

2011

2. **(a)** Verify that the point (2, –4) is on the line $3x - y = 10$.

(b) $P(2, 8)$, $Q(4, -1)$ and $R(6, 0)$ are three points.

(i) Find the slope of PR.

(ii) Show that PR is perpendicular to RQ.

(iii) Find the equation of RQ.

(iv) Find the co-ordinates of the point at which RQ intersects the y-axis.

(c) $A(-1, -6)$, $B(6, 8)$ and $C(2, 5)$ are three points.

(i) Find the area of the triangle ABC.

(ii) Find the co-ordinates of two possible points D on the x-axis such that area of triangle ABD = area of triangle ABC.

Solution

2 (a)

> **Is a point on a line?**
> To show a point is on a line, put the point into the equation.

$$(2,\ -4) \in 3x - y = 10\,?$$
$$3(2) - (-4)$$
$$= 6 + 4$$
$$= 10$$

2 (b) (i)

$P(2, 8)$, $Q(4, -1)$, $R(6, 0)$

Slope of PR:

$$\boxed{\begin{array}{cc} P(2,\ 8) & R(6,\ 0) \\ \downarrow\downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ \ y_2 \end{array}}$$

Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-8}{6-2} = \dfrac{-8}{4} = -2$

2 (b) (ii)

$P(2, 8)$, $Q(4, -1)$, $R(6, 0)$

Slope of RQ:

$$\boxed{\begin{array}{cc} R(6,\ 0) & Q(4,\ -1) \\ \downarrow\downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ \ y_2 \end{array}}$$

Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-0}{4-6} = \dfrac{-1}{-2} = \dfrac{1}{2}$

Two lines are perpendicular if the product of their slopes is -1.

$-2\times\frac{1}{2}=-1$ [Therefore, *PR* is perpendicular to *RQ*.]

2 (b) (iii)

Equation of *RQ*:

$m=\frac{1}{2},\ (x_1,\ y_1)=R(6,\ 0)$

$y-0=\frac{1}{2}(x-6)$

Equation of a line: $\boxed{y-y_1=m(x-x_1)}$

$2y=1(x-6)$

$2y=x-6$

$x-2y-6=0$

2 (b) (iv)

To find the *x*-intercept: Put $y=0$.
To find the *y*-intercept: Put $x=0$.

$$x=0\Rightarrow(0)-2y-6=0$$
$$-2y=6$$
$$y=-3$$

Therefore, $(0,-3)$ is the *y* intercept.

2 (c) (i)

$A(-1,-6),\ B(6,8),\ C(2,5)$

$\boxed{A=\frac{1}{2}|x_1y_2-x_2y_1|}$

Steps
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

Area *A* of triangle *ABC*:

$A(-1,\ -6)\ \rightarrow\ (0,\ 0)$

$B(6,\ 8)\ \rightarrow\ (7,\ 14)$

$C(2,\ 5)\ \rightarrow\ (3,\ 11)$

$$\begin{array}{cccc} (7, & 14) & (3, & 11) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\begin{aligned} A&=\tfrac{1}{2}|(7)(11)-(3)(14)| \\ &=\tfrac{1}{2}|77-42| \\ &=\tfrac{1}{2}|35| \\ &=\tfrac{35}{2}\text{ square units} \end{aligned}$$

2 (c) (ii)

Point D is on the y-axis. Therefore, its co-ordinates are $(x, 0)$

Area A of triangle ABD:

$$
\begin{array}{lcl}
A(-1, -6) & \rightarrow & (0, 0) \\
B(6, 8) & \rightarrow & (7, 14) \\
D(x, 0) & \rightarrow & (x+1, 6)
\end{array}
$$

$$\boxed{\begin{array}{cccc} (7, & 14) & (x+1, & 6) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}}$$

Area of ABD = Area of ABC

$$\tfrac{1}{2}\left|(7)(6)-(14)(x+1)\right| = \tfrac{35}{2}$$

$$\left|(7)(6)-(14)(x+1)\right| = 35$$

$$\left|42-14x-14\right| = 35$$

$$\left|28-14x\right| = 35$$

$$28-14x = \pm 35$$

Solve each equation separately:

$28-14x=35$	$28-14x=-35$
$28-35=14x$	$28+35=14x$
$-7=14x$	$63=14x$
$-\frac{7}{14}=x$	$\frac{63}{14}=x$
$\therefore x=-\frac{1}{2}$	$\therefore x=\frac{9}{2}$

Answer: $D(-\tfrac{1}{2}, 0), (\tfrac{9}{2}, 0)$