

THE LINE (Q 2, PAPER 2)

2010

2 (a) Find the area of the triangle with vertices $(0, 0)$, $(8, -6)$ and $(-1, 5)$.

(b) l is the line $3x - 4y - 15 = 0$.

(i) Verify that $(1, -3)$ is a point on l .

(ii) l intersects the x -axis at P . Find the co-ordinates of P .

The line k passes through the point $(1, -3)$ and is perpendicular to l .

(iii) Show the lines l and k on a co-ordinate diagram.

(iv) Find the equation of k .

(c) $A(2, -1)$ and $B(-4, 7)$ are two points.

(i) Find $|AB|$.

(ii) Find C , the image of B under the translation $(2, -1) \rightarrow (-7, 11)$.

(iii) Show that $|AB|:|AC| = 2:5$.

SOLUTION

2 (a)

$$\begin{aligned} A &= \frac{1}{2} |(8)(5) - (-1)(-6)| \\ &= \frac{1}{2} |(40) - (6)| \\ &= \frac{1}{2} |34| = 17 \text{ units squared} \end{aligned}$$

$$A = \frac{1}{2} |x_1 y_2 - x_2 y_1|$$

$A(8, -6)$	$B(-1, 5)$
$\downarrow \quad \downarrow$	$\downarrow \quad \downarrow$
$x_1 \quad y_1$	$x_2 \quad y_2$

2 (b) (i)

$$(1, -3) \in l?$$

$$3(1) - 4(-3) - 15$$

$$= 3 + 12 - 15$$

$$= 0 \Rightarrow (1, -3) \in l$$

IS A POINT ON A LINE?

To show a point is on a line, put the point into the equation.

2 (b) (ii)

$$y = 0: 3x - 4(0) - 15 = 0$$

$$3x - 15 = 0$$

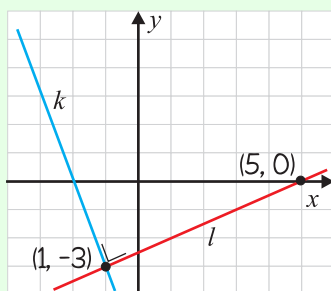
$$3x = 15$$

$$x = 5$$

$$\therefore P(5, 0)$$

To find the x -intercept: Put $y = 0$.

To find the y -intercept: Put $x = 0$.

2 (b) (iii)**2 (b) (iv)**

Slope of l : $m = \frac{3}{4}$

Slope of k : $m = -\frac{4}{3}$

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

Equation of k : Point $(1, -3)$, $m = -\frac{4}{3}$

$$y - (-3) = -\frac{4}{3}(x - 1)$$

$$y + 3 = -\frac{4}{3}(x - 1)$$

$$3(y + 3) = -4(x - 1)$$

$$3y + 9 = -4x + 4$$

$$4x + 3y + 5 = 0$$

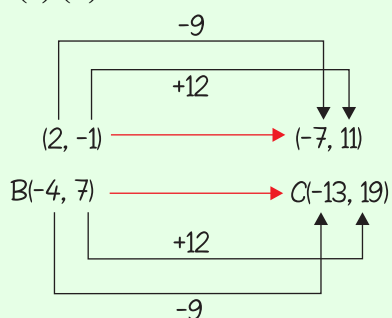
$$y - y_1 = m(x - x_1)$$

2 (c) (i)

$$\begin{aligned} |AB| &= \sqrt{(-4 - 2)^2 + (7 - (-1))^2} \\ &= \sqrt{(-6)^2 + (8)^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} = 10 \end{aligned}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$A(2, -1)$	$B(-4, 7)$
$\downarrow \downarrow$	$\downarrow \downarrow$
$x_1 \quad y_1$	$x_2 \quad y_2$

2 (c) (ii)**2 (c) (ii)**

$$\begin{aligned} |AC| &= \sqrt{(-13 - 2)^2 + (19 - (-1))^2} \\ &= \sqrt{(-15)^2 + (20)^2} \\ &= \sqrt{225 + 400} \\ &= \sqrt{625} = 25 \end{aligned}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$A(2, -1)$	$C(-13, 19)$
$\downarrow \downarrow$	$\downarrow \downarrow$
$x_1 \quad y_1$	$x_2 \quad y_2$

$$|AB| : |AC| = 10 : 25 = 2 : 5$$