## The Line (Q 2, Paper 2)

2009
2 (a) $a(-2,1)$ and $b(4,5)$ are two points.
(i) Plot the points $a$ and $b$ on a co-ordinate diagram.
(ii) Find the slope of $a b$.
(iii) Find the equation of $a b$.
$K$ is the line $3 x+2 y-9=0$.
(iv) Show that $K$ passes through the midpoint of [ab].
(v) Show that $K$ is perpendicular to $a b$.
(b) $p(3,0)$ is a point.
$t$ and $s$ are two distinct points on the $y$-axis and $|p t|=|p s|=5$.
(i) Find the co-ordinates of $t$ and the co-ordinates of $s$.
(ii) Find the area of the triangle tsp.
(iii) ptus is a parallelogram in which [ts] is a diagonal.

Find the co-ordinates of the point $u$.

## Solution

2 (a) (i)


## 2 (a) (iii)

Point ( $-2,1$ ), $m=\frac{2}{3}$
$y-1=\frac{2}{3}(x-(-2)) \quad y-y_{1}=m\left(x-x_{1}\right)$
$y-1=\frac{2}{3}(x+2)$
$3(y-1)=2(x+2)$
$3 y-3=2 x+4$
$0=2 x-3 y+7$

$$
\begin{gathered}
\text { 2 (a) (ii) } \begin{array}{ccc}
\begin{array}{ccc}
a(-2,1) & b(4,5) \\
\downarrow & \downarrow & \downarrow \downarrow \\
x_{1} & y_{1} & x_{2}
\end{array} y_{2}
\end{array} \quad m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
m=\frac{5-1}{4-(-2)}=\frac{4}{6}=\frac{2}{3}
\end{gathered}
$$

2 (a) (iv)

$$
\text { Midpoint }=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)
$$

Midpoint $=\left(\frac{-2+4}{2}, \frac{1+5}{2}\right)=\left(\frac{2}{2}, \frac{6}{2}\right)=(1,3)$

## Is a point on a line?

To show a point is on a line, put the point into the equation.

$$
\begin{aligned}
& (1,3) \in K ? \\
& 3(1)+2(3)-9=3+6-9=0 \\
& \Rightarrow(1,3) \in K
\end{aligned}
$$

## 2 (a) (v)

Slope of $K: 3 x+2 y-9=0 \Rightarrow m=-\frac{3}{2}$

$$
m=-\left(\frac{\text { Number in front of } x}{\text { Number in front of } y}\right)
$$

Slope of $a b: m=\frac{3}{2}$
$-\frac{3}{2} \times \frac{2}{3}=-1$ [Therefore, $K$ is perpendicular to $a b$.]

> Two lines are perpendicular if the product of their slopes is -1 .

$$
K \perp L \Rightarrow m_{1} \times m_{2}=-1
$$

## 2 (b) (i)

Sketch the diagram. You have 2 right-angled triangles so you can apply Pythagoras.


$$
\begin{aligned}
& x^{2}+y^{2}=r^{2} \\
& 3^{2}+y^{2}=5^{2} \\
& 9+y^{2}=25 \\
& y^{2}=25-9=16 \\
& y=\sqrt{16}=4
\end{aligned}
$$



As you see the points are $t(0,4)$ and $s(0,-4)$.


## 2 (b) (ii)

There are 2 right-angled triangles that have the same area.
The area of each triangle is half the base by the height.
Area of $t s p=2 \times \frac{1}{2} \times 3 \times 4=12$

2 (b) (iii)


The fourth point is found by passing the point $p$ through the origin by a central symmetry. $p(3,0) \rightarrow(0,0) \rightarrow u(-3,0)$


