THE LINE (Q 2, PAPER 2)

2007

2 (a) Find the co-ordinates of the mid-point of the line segment joining the points $(2, -3)$ and $(6, 9)$.

(b) The line L intersects the x-axis at $(-4, 0)$ and the y-axis at $(0, 6)$.

(i) Find the slope of L.

(ii) Find the equation of L.

The line K passes through $(0, 0)$ and is perpendicular to L.

(iii) Show the lines L and K on a co-ordinate diagram.

(iv) Find the equation of K.

(c) $a(-4, 3)$, $b(6, -1)$ and $c(2, 7)$ are three points.

(i) Find the area of triangle abc.

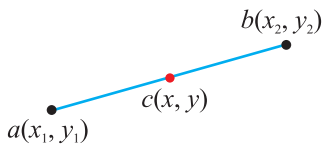
(ii) $abcd$ is a parallelogram in which $[ac]$ is a diagonal.
Find the co-ordinates of the point d.

SOLUTION

2 (a)

The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\boxed{\text{Midpoint } = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)} \ldots\ldots\ 2$$



REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint } = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2}\right)$

$$a(2, -3) \quad b(6, 9)$$
$$\downarrow \ \ \downarrow \qquad \downarrow \ \ \downarrow$$
$$x_1 \ \ y_1 \qquad x_2 \ \ y_2$$

$$\text{Midpoint } = \left(\frac{2+6}{2}, \frac{-3+9}{2}\right) = \left(\frac{8}{2}, \frac{6}{2}\right) = (4, 3)$$

2 (b) (i)

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}} \ldots\ldots\ 3$$

REMEMBER IT AS:

$$\text{Slope } m = \frac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$$

$$a(-4, 0) \quad b(0, 6)$$
$$\downarrow \ \ \downarrow \qquad \downarrow \ \ \downarrow$$
$$x_1 \ \ y_1 \qquad x_2 \ \ y_2$$

$$m = \frac{6-0}{0-(-4)} = \frac{6}{4} = \frac{3}{2}$$

2 (b) (ii)

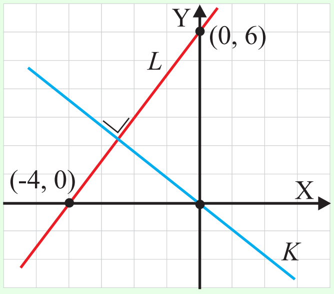
The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ 4

Slope $m = \frac{3}{2}$, given point $(x_1, y_1) = (-4, 0)$.

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - (0) = \tfrac{3}{2}(x - (-4))$$
$$\Rightarrow y = \tfrac{3}{2}(x + 4)$$
$$\Rightarrow 2y = 3(x + 4)$$
$$\Rightarrow 2y = 3x + 12$$
$$\therefore 3x - 2y + 12 = 0$$

2 (b) (iii)



2 (b) (iv)

Finding the perpendicular slope: Invert the slope and change its sign.

Equation of K: The slope is perpendicular to L. $\therefore m = -\frac{2}{3}$. Point $(x_1, y_1) = (0, 0)$.

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 0 = -\tfrac{2}{3}(x - 0)$$
$$\Rightarrow 3y = -2x$$
$$\therefore 2x + 3y = 0$$

2 (c) (i)

The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{A = \frac{1}{2}|x_1 y_2 - x_2 y_1|}$ 6

Steps
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

$a(-4, 3) \rightarrow (0, 0)$

$b(6, -1) \rightarrow (10, -4)$

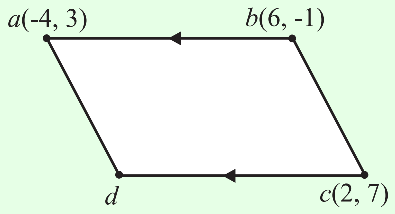
$c(2, 7) \rightarrow (6, 4)$

Translate a to (0, 0) by adding 4 to the x part and taking 3 away from the y part. Do the same to the other two points.

$$\boxed{\begin{array}{cccc} \multicolumn{2}{c}{a(10, -4)} & \multicolumn{2}{c}{b(6, 4)} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}}$$

$$A = \tfrac{1}{2}|(10)(4) - (-4)(6)|$$
$$\Rightarrow A = \tfrac{1}{2}|40 + 24|$$
$$\therefore A = \tfrac{1}{2}|64| = 32 \text{ square units}$$

2 (c) (ii) Sketch the points *abc*. The fourth point, *d*, of the parallelogram *abcd* is obtained by finding the image of *c* under a translation from *b* to *a*.



$$b(6,\,-1)\ \rightarrow a(-4,\,3)$$
$$c(2,\,7)\ \ \rightarrow d(-8,\,11)$$