## The Line (Q 2, Paper 2)

2006

2 (a) $a(-2,6)$ and $b(4,3)$ are two points.
(i) Plot $a$ and $b$ on a co-ordinate diagram.
(ii) From your diagram, write down the co-ordinates of the point at which the line $a b$ cuts the $y$-axis.
(iii) Find the slope of $a b$.
(iv) Calculate the area of the triangle $a b c$, where the co-ordinates of $c$ are $(1,-3)$.
(b) $L$ is the line $3 x+2 y+c=0$.
(i) $(3,-1)$ is a point on $L$. Find the value of $c$.
(ii) The line $K$ is parallel to $L$ and passes through the point $(-2,5)$.

Find the equation of $K$.
(iii) The lines $L$ and $K$, together with the line $x=3$ and the $y$-axis, form a parallelogram. Find the co-ordinates of the vertices of the parallelogram.
Solution
2 (a) (i)


2 (a) (ii)
You can see the line cuts the $y$-axis at $(0,5)$.
2 (a) (iii)

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \ldots \ldots .3 \quad \begin{aligned}
& \text { Remember it as: } \\
& \text { Slope } m=\frac{\text { Difference in } y^{\prime} s}{\text { Difference in } x^{\prime} \text { 's }}
\end{aligned}
$$

$$
\begin{array}{cc}
a(-2,6) & b(4, \\
\downarrow & \downarrow \\
x_{1} & y_{1} \\
& x_{2} \\
\hline
\end{array}
$$

$$
m=\frac{3-6}{4-(-2)}=\frac{-3}{4+2}=\frac{-3}{6}=-\frac{1}{2}
$$

2 (a) (iv) The area, $A$, of $\Delta a o b$ with vertices $o(0,0), a\left(x_{1}, y_{1}\right), b\left(x_{2}, y_{2}\right)$ is given by:

$$
A=\frac{1}{2}\left|x_{1} y_{2}-x_{2} y_{1}\right|
$$

6

## Steps

1. Translate one point to $(0,0)$.
2. Do the same translation to the other two points.
3. Apply the formula.

$$
\begin{array}{lll}
\begin{array}{lll}
a(-2,6) & \rightarrow & (-3,9) \\
b(4,3) & \rightarrow & (3,6)
\end{array} & \begin{array}{l}
\text { Translate } c \text { to }(0,0) \text { by taking } \\
\text { adding } 3 \text { to the } y \text { part. Do the } s \\
\text { points. }
\end{array} \\
\begin{array}{ccc}
\begin{array}{ccc}
a(-3,-3) & \rightarrow & (0,0)
\end{array} & b(3,6) \\
\downarrow & \downarrow & \downarrow \\
x_{1} & y_{1} & x_{2} \\
y_{2}
\end{array} & \begin{array}{l}
A=\frac{1}{2}|(-3)(6)-(9)(3)| \\
\Rightarrow A=\frac{1}{2}|-18-27| \\
\therefore A=\frac{1}{2}|-45|=\frac{45}{2} \text { square units }
\end{array}
\end{array}
$$

Translate $c$ to $(0,0)$ by taking 1 away from the $x$ part and adding 3 to the $y$ part. Do the same to the other two

2 (b) (i)
Is a point on a line?
To show a point is on a line, put the point into the equation.
You are told that $(3,-1)$ is on $L$. Therefore, you can substitute it in to find $c$. Replace $x$ by 3 and $y$ by -1 .
$(3,-1) \in L \Rightarrow 3(3)+2(-1)+c=0$
$\Rightarrow 9-2+c=0$
$\Rightarrow 7+c=0$
$\therefore c=-7$
2 (b) (ii)

## Parallel lines have the same slope.

$$
K \| L \Rightarrow m_{1}=m_{2}
$$

Equation of $L: 3 x+2 y-7=0$


$$
\text { Slope: } m=-\left(\frac{a}{b}\right) \text {....... } 5 \text { Remember it as: Slope } m=-\left(\frac{\text { Number in front of } x}{\text { Number in front of } y}\right)
$$

Slope of $L: m=-\frac{3}{2}$
Therefore, slope of $K$ : $m=-\frac{3}{2}$
Equation of $K$ : slope $m=-\frac{3}{2}$, point $\left(x_{1}, y_{1}\right)=(-2,5)$.
$\left(x_{1}, y_{1}\right)=(-2,5)$.
$y-y_{1}=m\left(x-x_{1}\right)$
$\Rightarrow y-5=-\frac{3}{2}(x-(-2))$
$\Rightarrow 2(y-5)=-3(x+2)$
$\Rightarrow 2 y-10=-3 x-6$
$\therefore 3 x+2 y-4=0$

## 2 (b) (ii)

The $y$-axis has an equation $x=0$. This is parallel to the line $x=3$. You need to find out where the lines $L$ and $K$ intersect these lines.

Points of intersection of $L: 3 x+2 y-7=0$ with: $x=0$ :
$3(0)+2 y-7=0 \Rightarrow 2 y=7$
$\Rightarrow y=\frac{7}{2} \Rightarrow\left(0, \frac{7}{2}\right)$ is a point of intersection.
$x=3$ :
3(3) $+2 y-7=0 \Rightarrow 9+2 y=7$
$\Rightarrow 2 y=-2 \Rightarrow y=-1 \Rightarrow(0,-1)$ is a point of intersection.

Points of intersection of $K: 3 x+2 y-4=0$ with:
$x=0$ :


Vertical lines have equations where $x=$ constant. In particular, the $y$-axis has the equation $x=0$.
$3(0)+2 y-4=0 \Rightarrow 0+2 y=4$
$\Rightarrow y=2 \Rightarrow(0,2)$ is a point of intersection.
$x=3$ :
3(3) $+2 y-4=0 \Rightarrow 9+2 y=4$
$\Rightarrow 2 y=-5 \Rightarrow y=-\frac{5}{2} \Rightarrow\left(3,-\frac{5}{2}\right)$ is a point of intersection.
The co-ordinates of the vertices of the parallelogram are: $(0,2),\left(3,-\frac{5}{2}\right),\left(0, \frac{7}{2}\right),(3,-1)$

