THE LINE (Q 2, PAPER 2)

2005

2 (a) Find the distance between the two points (3, 4) and (15, 9).

(b) L is the line $3x-4y+12=0$.
L intersects the x-axis at a and the y-axis at b.
(i) Find the co-ordinates of a and the co-ordinates of b.

(ii) K is the line that passes through b and is perpendicular to L.
Show L and K on a co-ordinate diagram.

(iii) Find the equation of K.

(iv) The point $(2t, 3t)$ is on the line K. Find the value of t.

(c) The lines $2x-y+3=0$ and $4x-y+k=0$ intersect at a point.
(i) Find, in terms of k, the co-ordinates of the point of intersection of the lines.

(ii) For what value of k is the point of intersection on the y-axis?

SOLUTION

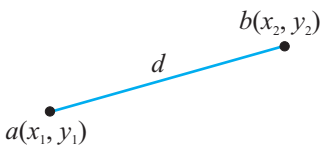
2 (a)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \quad \ldots\ldots \ (1)$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d=\sqrt{(\text{Difference in } x\text{'s})^2+(\text{Difference in } y\text{'s})^2}$$



$a(3, 4)$ $b(15, 9)$
$x_1\ y_1$ $x_2\ y_2$

$$|ab|=\sqrt{(15-3)^2+(9-4)^2}$$
$$\Rightarrow |ab|=\sqrt{12^2+5^2}=\sqrt{144+25}$$
$$\therefore \Rightarrow |ab|=\sqrt{169}=13$$

2 (b) (i)

To find the x-intercept: Put $y = 0$.
To find the y-intercept: Put $x = 0$.

x-intercept: Put $y = 0$

$\Rightarrow 3x-4(0)+12=0 \Rightarrow 3x=-12$

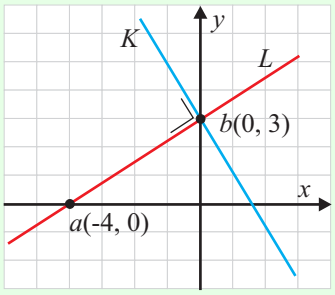
$\therefore x=-4 \Rightarrow a(-4, 0)$ is the x-intercept.

y-intercept: Put $x = 0$

$\Rightarrow 3(0)-4y+12=0 \Rightarrow 12=4y$

$\therefore y=3 \Rightarrow b(0, 3)$ is the y-intercept.

2 (b) (ii)



2 (b) (iii)

You need to find the slope of L first.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \ldots\ldots \textbf{3}$$

REMEMBER IT AS:

Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cc} a(-4,\ 0) & b(0,\ 3) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ \ y_1 & x_2\ \ y_2 \end{array}$$

$$m = \frac{3-0}{0-(-4)} = \frac{3}{4}$$

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

K is perpendicular to L.

Therefore, slope of L: $m = -\frac{4}{3}$

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $y - y_1 = m(x - x_1)$ **4**

Equation of K: Point $(x_1, y_1) = (0, 3)$, $m = -\frac{4}{3}$

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 3 = -\tfrac{4}{3}(x - 0)$$
$$\Rightarrow 3(y - 3) = -4x$$
$$\Rightarrow 3y - 9 = -4x$$
$$\therefore 4x + 3y - 9 = 0$$

2 (b) (iv)

IS A POINT ON A LINE?
To show a point is on a line, put the point into the equation.

$$(2t,\ 3t) \in K \Rightarrow 4(2t) + 3(3t) - 9 = 0$$
$$\Rightarrow 8t + 9t = 9$$
$$\Rightarrow 17t = 9$$
$$\therefore t = \tfrac{9}{17}$$

2 (c) (i)

INTERSECTION LINES
To find out where two lines intersect, solve their equations **simultaneously**.

$$2x - y + 3 = 0 \ldots\ldots (\mathbf{1}) (\times -1)$$
$$4x - y + k = 0 \ldots\ldots (\mathbf{2})$$

$$\begin{array}{r} -2x + y - 3 = 0 \\ 4x - y + k = 0 \\ \hline 2x \quad + (k-3) = 0 \end{array}$$

$$\Rightarrow 2x = -(k-3)$$
$$\Rightarrow 2x = 3 - k$$
$$\therefore x = \frac{3-k}{2}$$

Substitute this value of x back into Eqn. (**1**).

$$2x - y + 3 = 0 \Rightarrow 2\left(\frac{3-k}{2}\right) - y + 3 = 0$$
$$\Rightarrow 3 - k - y + 3 = 0$$
$$\Rightarrow 6 - k = y$$

ANS: $(x,\ y) = \left(\frac{3-k}{2},\ 6-k\right)$

2 (c) (ii)

If the point of intersection is on the y-axis, the x part must be zero.

$$\therefore \frac{3-k}{2} = 0 \Rightarrow 3 - k = 0$$
$$\therefore k = 3$$