THE LINE (Q 2, PAPER 2)

2004

2 (a) $p(5, -8)$ and $q(11, 10)$ are two points.
Find the co-ordinates of the midpoint of $[pq]$.

(b) $a(-1, -2)$, $b(3, 1)$, $c(0, 4)$ are three points.

(i) Find the length of $[ab]$.

(ii) Calculate the area of the triangle abc.

(iii) The line L is parallel to ab and passes through the point c.
Find the equation of L.

(iv) Show that the point $d(-4, 1)$ is on L.

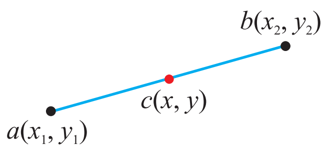
(v) Investigate whether $abcd$ is a parallelogram.

SOLUTION

2 (a)

The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\boxed{\text{Midpoint } = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)} \text{ .... } 2$$



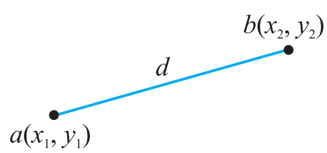
REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint } = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2}\right)$

$$\begin{array}{cccc} p(5, & -8) & q(11, & 10) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\text{Midpoint } = \left(\frac{5+11}{2}, \frac{-8+10}{2}\right) = \left(\frac{16}{2}, \frac{2}{2}\right) = (8, 1)$$

2 (b) (i)

$$\boxed{d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}} \text{ .... } 1$$



The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$

$$\begin{array}{cccc} a(-1, & -2) & b(3, & 1) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$|ab| = \sqrt{(3-(-1))^2+(1-(-2))^2}$$

$$\Rightarrow |ab| = \sqrt{(3+1)^2+(1+2)^2}$$

$$\Rightarrow |ab| = \sqrt{4^2+3^2} = \sqrt{16+9}$$

$$\therefore |ab| = \sqrt{25} = 5$$

2 (b) (ii) The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{A = \frac{1}{2}|x_1y_2 - x_2y_1|}$ **6**

STEPS
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

$a(-1, -2) \rightarrow (0, 0)$
$b(3, 1) \rightarrow (4, 3)$
$c(0, 4) \rightarrow (1, 6)$

Translate a to (0, 0) by adding 1 to the x part and adding 2 to the y part. Do the same to the other two points.

$$\begin{array}{cc} (4, 3) & (1, 6) \\ \downarrow\downarrow & \downarrow\downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$A = \frac{1}{2}|(4)(6) - (3)(1)|$

$\Rightarrow A = \frac{1}{2}|24 - 3| = \frac{1}{2}|21|$

$\therefore A = \frac{21}{2}$ square units

2 (b) (iii)
Firstly, find the slope of ab.

$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$ **3**

REMEMBER IT AS:

Slope $m = \frac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cc} a(-1, -2) & b(3, 1) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ \ y_1 & x_2\ y_2 \end{array}$$

$$m = \frac{1-(-2)}{3-(-1)} = \frac{1+2}{3+1} = \frac{3}{4}$$

Parallel lines have the same slope.

The slope of L is also $\frac{3}{4}$.

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ **4**

Equation of L: Point $(x_1, y_1) = (0, 4)$, slope $m = \frac{3}{4}$.

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 4 = \frac{3}{4}(x - 0)$

$\Rightarrow 4(y - 4) = 3x$

$\Rightarrow 4y - 16 = 3x$

$\therefore 3x - 4y + 16 = 0$

2 (b) (iv)

IS A POINT ON A LINE?
To show a point is on a line, put the point into the equation.

If you put the point d into L, it will satisfy the equation.

$d(-4, 1) \in L$? $3(-4) - 4(1) + 16 = -12 - 4 + 16$

$= 0 \Rightarrow d(-4, 1) \in L$

2 (b) (v)

To prove *abcd* is a parallelogram: Find the slopes of each side and show opposite sides have the same slopes.

Slope of *ab*: $m = \frac{3}{4}$

Slope of *dc*: $m = \dfrac{1-4}{-4-0} = \dfrac{-3}{-4} = \dfrac{3}{4}$

Slope of *ad*: $m = \dfrac{1-(-2)}{-4-(-1)} = \dfrac{1+2}{-4+1} = \dfrac{3}{-3} = -1$

Slope of *bc*: $m = \dfrac{1-4}{3-0} = \dfrac{-3}{3} = -1$

Opposite sides have the same slope. Therefore, *abcd* is a parallelogram.