THE LINE (Q 2, PAPER 2)

2003

2 (a) Find the distance between the two points (3, 2) and (8, 14).

(b) $a(-2, 2)$, $b(4, 6)$ and $c(0, -4)$ are three points.
p is the midpoint of $[ab]$ and q is the midpoint of $[ac]$.
(i) Find the co-ordinates of p and the co-ordinates of q.

(ii) Plot a, b, c, p and q on a co-ordinate diagram on graph paper.
Show the line segments $[bc]$ and $[pq]$ on your diagram.

(iii) Using slopes, or otherwise, prove that pq is parallel to bc.

(c) L is the line $3x + 2y + 12 = 0$.
K is the line that passes through the point (7, 3) and is perpendicular to L.
Find the equation of K and hence find the point of intersection of K and L.

SOLUTION

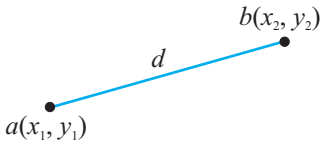
2 (a)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots \textbf{1}$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



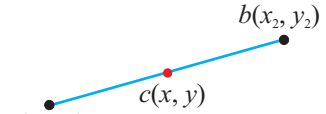
$a(3, 2)$ $b(8, 14)$
$x_1\ y_1$ $x_2\ y_2$

$$|ab| = \sqrt{(8-3)^2 + (14-2)^2}$$
$$\Rightarrow |ab| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144}$$
$$\therefore \Rightarrow |ab| = \sqrt{169} = 13$$

2 (b) (i)

The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \ldots\ldots \textbf{2}$$



REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint} = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2}\right)$

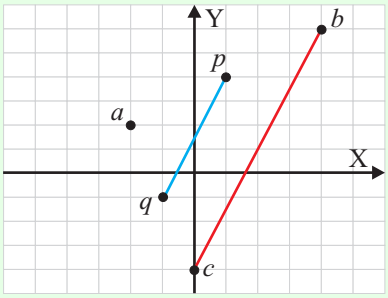
$a(-2, 2)$ $b(4, 6)$
$x_1\ y_1$ $x_2\ y_2$

$$\text{Midpoint of } [ab] = \left(\frac{-2+4}{2}, \frac{2+6}{2}\right) = \left(\frac{2}{2}, \frac{8}{2}\right) = p(1, 4)$$

$a(-2, 2)$ $c(0, -4)$
$x_1\ y_1$ $x_2\ y_2$

$$\text{Midpoint of } [ac] = \left(\frac{-2+0}{2}, \frac{2-4}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right) = q(-1, -1)$$

2 (b) (ii)



2 (b) (iii)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \ldots\ldots \; 3$$

REMEMBER IT AS:

Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

Parallel lines have the same slope.

$$\begin{matrix} b(4,\ 6) & c(0,\ -4) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ \ y_1 & x_2\ \ y_2 \end{matrix}$$

Slope of bc: $m = \dfrac{-4-6}{0-4} = \dfrac{-10}{-4} = \dfrac{5}{2}$

$$\begin{matrix} p(1,\ 4) & q(-1,\ -4) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ \ y_1 & x_2\ \ y_2 \end{matrix}$$

Slope of pq: $m = \dfrac{-1-4}{-1-1} = \dfrac{-5}{-2} = \dfrac{5}{2}$

As the slope of bc and pq are equal, lines pq and ab are parallel.

2 (c)

GENERAL FORM OF A STRAIGHT LINE

Every straight line can be written in the form: $ax + by + c = 0$.

You can read off the slope of a straight line from its equation.

Slope: $m = -\left(\dfrac{a}{b}\right)$ 5

REMEMBER IT AS: Slope $m = -\left(\dfrac{\text{Number in front of } x}{\text{Number in front of } y}\right)$

L: $3x + 2y + 12 = 0$

Slope of L: $m = -\frac{3}{2}$

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $y - y_1 = m(x - x_1)$ 4

Equation of K: Slope $m = \frac{2}{3}$, point $(x_1,\ y_1) = (7,\ 3)$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = \frac{2}{3}(x - 7)$

$\Rightarrow 3(y - 3) = 2(x - 7)$

$\Rightarrow 3y - 9 = 2x - 14$

$\therefore 2x - 3y - 5 = 0$

INTERSECTING LINES
To find out where two lines intersect, solve their equations **simultaneously**.

$3x + 2y + 12 = 0 \ldots..(\mathbf{1})\ (\times 3)$
$2x - 3y - 5 = 0 \ldots..(\mathbf{2})\ (\times 2)$

$\rightarrow$

$9x + 6y + 36 = 0$
$4x - 6y - 10 = 0$
$13x + 26 = 0 \Rightarrow 13x = -26 \Rightarrow x = -2$

Substitute this value for x into Eqn. (**1**).

$3(-2) + 2y + 12 = 0 \Rightarrow -6 + 2y + 12 = 0$

$\Rightarrow 2y + 6 = 0$

$\Rightarrow 2y = -6$

$\therefore y = -3$

Point of intersection: $(-2, -3)$