## The Line (Q 2, Paper 2)

2002
2 (a) Find the co-ordinates of the point of intersection of the line and the line $4 x+y=5$ and $3 x-2 y=12$.
(b) The line $L$ has equation $4 x-5 y=-40$. $a(0,8)$ and $b(-10,0)$ are two points.
(i) Verify that $a$ and $b$ lie on $L$.
(ii) What is the slope of $L$ ?
(iii) The line $K$ is perpendicular to $L$ and it contains $b$. Find the equation of $K$.
(iv) $K$ intersects the $y$-axis at the point $c$. Find the co-ordinates of $c$.
(v) $d$ is another point such that $a b c d$ is a rectangle. Calculate the area of $a b c d$.
(vi) Find the co-ordinates of $d$.

## Solution

2 (a)

## Intersecting Lines

To find out where two lines intersect, solve their equations simultaneously.

$$
\begin{aligned}
& 4 x+y=5 \ldots . .(\mathbf{1})(\times 2) \\
& 3 x-2 y=12 \ldots(2)
\end{aligned} \longrightarrow \begin{aligned}
& 8 x+2 y=10 \\
& \frac{3 x-2 y=12}{11 x}=22 \Rightarrow x=2
\end{aligned}
$$

Substitute this value of $x$ into Eqn. (1).
$\therefore 4(2)+y=5 \Rightarrow 8+y=5$
$\therefore y=-3$
Point of intersection: $(2,-3)$
2 (b) (i)
Is A point on a line?
To show a point is on a line, put the point into the equation.
$a(0,8) \in L$ ?
$4 x-5 y=4(0)-5(8)$
$=0-40$
$=-40 \Rightarrow a(0,8) \in L$
$b(-10,0) \in L$ ?
$4 x-5 y=4(-10)-5(0)$
$=-40+0$
$=-40 \Rightarrow b(-10,0) \in L$

## 2 (b) (ii)

## General form of a straight line

Every straight line can be written in the form: $a x+b y+c=0$.
You can read off the slope of a straight line from its equation.

$$
\text { Slope: } m=-\left(\frac{a}{b}\right) \ldots . .
$$

Remember it as: Slope $m=-\left(\frac{\text { Number in front of } x}{\text { Number in front of } y}\right)$

Slope of $L: m=\frac{4}{5}$
2 (b) (iii)
Finding the perpendicular slope: Invert the slope and change its sign.

The equation of a line is a formula satisfied by every point $(x, y)$ on the line.
Equation of a line:

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

Equation of $K$ : Slope $m=-\frac{5}{4}$, point $\left(x_{1}, y_{1}\right)=(-10,0)$
$y-y_{1}=m\left(x-x_{1}\right)$
$\Rightarrow y-0=-\frac{5}{4}(x-(-10))$
$\Rightarrow y=-\frac{5}{4}(x+10)$
$\Rightarrow 4 y=-5(x+10)$
$\Rightarrow 4 y=-5 x-50$
$\therefore 5 x+4 y+50=0$
2 (b) (iv)
To find the $x$-intercept: Put $y=0$.
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Put $x=0$ :
$\therefore 5(0)+4 y+50=0$
$\Rightarrow 4 y=-50$
$\therefore y=-\frac{50}{4}=-\frac{25}{2}$
$\Rightarrow c\left(0,-\frac{25}{2}\right)$ is the $y$ intercept.

## 2 (b) (v)

The rectangle $a b c d$ is bisected by the diagonal $a c$. Find the area of triangle $a b c$ and double it to find the area of $a b c d$.


The area, $A$, of $\Delta a o b$ with vertices $o(0,0), a\left(x_{1}, y_{1}\right), b\left(x_{2}, y_{2}\right)$ is given by:

$$
A=\frac{1}{2}\left|x_{1} y_{2}-x_{2} y_{1}\right|
$$

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## Steps

1. Translate one point to $(0,0)$.
2. Do the same translation to the other two points.
3. Apply the formula.

$$
\begin{array}{ll}
a(0,8) & \rightarrow(0,0) \\
b(-10,0) & \rightarrow(-10,-8) \\
c\left(0,-\frac{25}{2}\right) & \rightarrow\left(0,-\frac{41}{2}\right)
\end{array}
$$

Translate $a$ to $(0,0)$ by leaving the $x$ part unchanged and taking 8 away from the $y$ part. Do the same to the other two points.

Area of triangle $a b c: \quad A=\frac{1}{2}\left|(-10)\left(-\frac{41}{2}\right)-(-8)(0)\right|$

$$
\Rightarrow A=\frac{1}{2}|205+0|=\frac{205}{2}
$$

Area of rectangle $a b c d$ : $A=205$ square units
2 (b) (vi)
To find $d$, do a translation.


