THE LINE (Q 2, PAPER 2)

2001

2 (a) The point $(t, 2t)$ lies on the line $3x + 2y + 7 = 0$.
Find the value of t.

(b) $a(4, 2)$, $b(-2, 0)$ and $c(0, 4)$ are three points.

(i) Prove that $ac \perp bc$.

(ii) Prove that $|ac| = |bc|$.

(iii) Calculate the area of the triangle bac.

(iv) The diagonals of the square $bahg$ intersect at c.
Find the co-ordinates of h and the co-ordinates of g.

(v) Find the equation of the line bc and show that h lies on this line.

SOLUTION

2 (a)

> **IS A POINT ON A LINE?**
> To show a point is on a line, put the point into the equation.

Replace x by t and y by $2t$.

$$(t,\ 2t) \in 3x + 2y + 7 = 0$$
$$\Rightarrow 3(t) + 2(2t) + 7 = 0$$
$$\Rightarrow 3t + 4t + 7 = 0$$
$$\Rightarrow 7t = -7$$
$$\therefore t = -1$$

2 (b) (i)

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}} \quad \ldots\ldots \quad 3$$

> **REMEMBER IT AS:**
> Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cc} a(4,\ 2) & c(0,\ 4) \\ \downarrow\downarrow & \downarrow\downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

Slope of ac: $m_1 = \dfrac{4-2}{0-4} = \dfrac{2}{-4} = -\dfrac{1}{2}$

$$\begin{array}{cc} b(-2,\ 0) & c(0,\ 4) \\ \downarrow\ \downarrow & \downarrow\downarrow \\ x_1\ \ y_1 & x_2\ y_2 \end{array}$$

Slope of bc: $m_2 = \dfrac{4-0}{0-(-2)} = \dfrac{4}{2} = 2$

> Two lines are perpendicular if the product of their slopes is -1.

$$m_1 \times m_2 = (-\tfrac{1}{2}) \times 2 = -1 \Rightarrow ac \perp bc$$

$$K \perp L \Rightarrow m_1 \times m_2 = -1$$

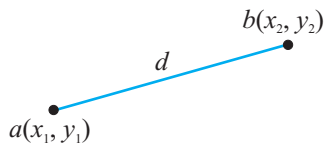
2 (b) (ii)

$$\boxed{d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}} \quad \ldots\ldots \; 1$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d=\sqrt{(\text{Difference in } x\text{'s})^2+(\text{Difference in } y\text{'s})^2}$$



$$\begin{array}{cc} a(4,\ 2) & c(0,\ 4) \\ \downarrow\downarrow & \downarrow\downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$$|ac|=\sqrt{(0-4)^2+(4-2)^2}$$
$$\Rightarrow |ac|=\sqrt{(-4)^2+(2)^2}=\sqrt{16+4}$$
$$\therefore |ac|=\sqrt{20}$$

$$\begin{array}{cc} b(-2,\ 0) & c(0,\ 4) \\ \downarrow\ \downarrow & \downarrow\downarrow \\ x_1\ \ y_1 & x_2\ y_2 \end{array}$$

$$|bc|=\sqrt{(0-(-2))^2+(4-0)^2}$$
$$\Rightarrow |bc|=\sqrt{(2)^2+(4)^2}=\sqrt{4+16}$$
$$\therefore |bc|=\sqrt{20}$$

$$\therefore |ac|=|bc|$$

2 (b) (iii) The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$$\boxed{A=\tfrac{1}{2}|x_1y_2-x_2y_1|} \quad \ldots\ldots \; 6$$

STEPS
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

$a(4,\ 2) \rightarrow (6,\ 2)$
$b(-2,\ 0) \rightarrow (0,\ 0)$
$c(0,\ 4) \rightarrow (2,\ 4)$

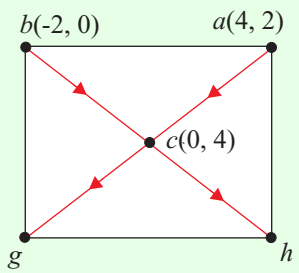
Translate b to (0, 0) by adding 2 to the x part and adding 0 to the y part. Do the same to the other two points.

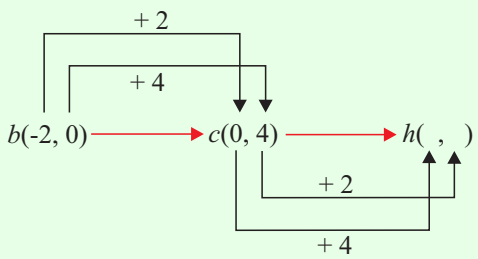
$$\begin{array}{cc} (6,\ 2) & (2,\ 4) \\ \downarrow\downarrow & \downarrow\downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$$A=\tfrac{1}{2}|(6)(4)-(2)(2)|$$
$$\Rightarrow A=\tfrac{1}{2}|24-4|=\tfrac{1}{2}|20|$$
$$\therefore A=10 \text{ square units}$$

2 (b) (iv)

The diagonals of a square bisect each other. To find out the co-ordinates of g and h send points b and a through c by a central symmetry.

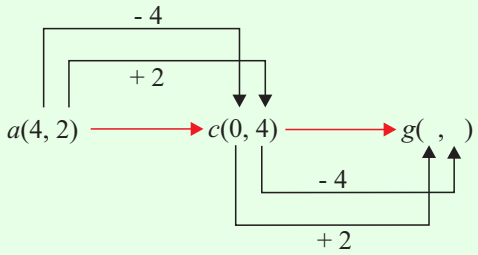




To go from b to c, you add 2 to the x-coordinate and 4 to the y-coordinate. Therefore, to go from c to h, you do exactly the same.

$\therefore b(-2,\ 0) \rightarrow c(0,\ 4) \rightarrow h(2,\ 8)$

NOTE: c is the midpoint of $[bh]$.



To go from a to c, you take away 4 from the x-coordinate and add 2 to the y-coordinate. Therefore, to go from c to g, you do exactly the same.

$\therefore a(4,\ 2) \rightarrow c(0,\ 4) \rightarrow g(-4,\ 6)$

Ans: $g(-4,\ 6)$, $h(2,\ 8)$

2 (b) (v)

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ **4**

Equation of bc: Point $c(0, 4)$, slope $m = 2$ [found in **2 (b) (i)**]

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 4 = 2(x - 0)$

$\Rightarrow y - 4 = 2x$

$\therefore 2x - y + 4 = 0$

Is a point on a line?
To show a point is on a line, put the point into the equation.

$h(2,\ 8) \in bc$?

$2(2) - (8) + 4$

$= 4 - 8 + 4$

$= 0 \Rightarrow h(2,\ 8) \in bc$