THE LINE (Q 2, PAPER 2)

2000

2 (a) Find the coordinates of the midpoint of the line segment which joins the points (2, – 3) and (– 8, – 6).

(b) $a(-2, -1)$, $b(1, 0)$ and $c(-5, 2)$ are three points.

(i) Show that $|ab| = \sqrt{10}$.

(ii) Find $|bc|$.

(iii) Hence, find the ratio $|ab| : |bc|$.
Give your answer in the form m:n where m and n are whole numbers.

(c) (i) The line L has equation $3x - 4y + 20 = 0$.
K is the line through $p(0, 5)$ which is perpendicular to L.
Find the equation of K.

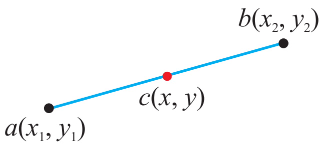
(ii) L cuts the x-axis at the point t.
K cuts the x-axis at the point r.
Calculate the area of the triangle ptr. Give your answer as a fraction.

SOLUTION

2 (a)

The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\boxed{\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)} \;\ldots\ldots\; \mathbf{2}$$



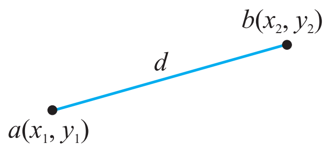
REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint} = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2}\right)$

$$\begin{array}{cccc} (2, & -3) & (-8, & -6) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\text{Midpoint} = \left(\frac{2-8}{2}, \frac{-3-6}{2}\right) = \left(\frac{-6}{2}, \frac{-9}{2}\right) = (-3, -\tfrac{9}{2})$$

2 (b) (i)

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \;\ldots\ldots\; \mathbf{1}$$



The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$

$$\begin{array}{cccc} a(-2, & -1) & b(1, & 0) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$|ab| = \sqrt{(1-(-2))^2 + (0-(-1))^2}$$

$$\Rightarrow |ab| = \sqrt{(1+2)^2 + (0+1)^2}$$

$$\Rightarrow |ab| = \sqrt{3^2 + 1^2} = \sqrt{9+1}$$

$$\therefore |ab| = \sqrt{10}$$

2 (b) (ii)

$$\begin{array}{cc} b(1,\ 0) & c(-5,\ 2) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$$|bc| = \sqrt{(-5-1)^2 + (2-0)^2}$$
$$\Rightarrow |bc| = \sqrt{(-6)^2 + (2)^2}$$
$$\Rightarrow |bc| = \sqrt{36+4}$$
$$\therefore |bc| = \sqrt{40} = \sqrt{4\times 10} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$$

2 (b) (iii)

$$|ab| : |bc| = \sqrt{10} : 2\sqrt{10} = 1:2$$

2 (c) (i)

General form of a straight line
Every straight line can be written in the form: $ax + by + c = 0$.
You can read off the slope of a straight line from its equation.

Slope: $\boxed{m = -\left(\frac{a}{b}\right)}$ 5

Remember it as: Slope $m = -\left(\frac{\text{Number in front of } x}{\text{Number in front of } y}\right)$

$$L: 3x - 4y + 20 = 0 \Rightarrow m = \tfrac{3}{4}$$

Finding the perpendicular slope: Invert the slope and change its sign.

K is perpendicular to L.

Equation of K: Point $p(0, 5)$, slope $m = -\frac{4}{3}$.

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ 4

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 5 = -\tfrac{4}{3}(x - 0)$$
$$\Rightarrow 3(y-5) = -4x$$
$$\Rightarrow 3y - 15 = -4x$$
$$\therefore 4x + 3y - 15 = 0$$

2 (c) (ii)

To find the x-intercept: Put $y = 0$.
To find the y-intercept: Put $x = 0$.

Finding where L cuts the x-axis.

$y = 0$: $3x - 4(0) + 20 = 0$

$\Rightarrow 3x = -20$

$\Rightarrow x = -\frac{20}{3} \Rightarrow (-\frac{20}{3},\ 0)$ is the x-intercept.

Finding where K cuts the x-axis.

$y = 0$: $4x + 3(0) - 15 = 0$

$\Rightarrow 4x = 15$

$\therefore x = \frac{15}{4} \Rightarrow (\frac{15}{4},\ 0)$ is the x-intercept.

The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{A = \frac{1}{2}|x_1y_2 - x_2y_1|}$ 6

Steps
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

$$\begin{aligned} p(0, 5) &\to (0, 0) \\ t(-\tfrac{20}{3}, 0) &\to (-\tfrac{20}{3}, -5) \\ r(\tfrac{15}{4}, 0) &\to (\tfrac{15}{4}, -5) \end{aligned}$$

Translate p to (0, 0) by adding 0 to the x part and taking 5 away from the y part. Do the same to the other two points.

$$\boxed{\begin{array}{cccc} (-\frac{20}{3}, & -5) & (\frac{15}{4}, & -5) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}}$$

$$A = \tfrac{1}{2}\left|(-\tfrac{20}{3})(-5) - (-5)(\tfrac{15}{4})\right|$$

$$\Rightarrow A = \tfrac{1}{2}\left|\tfrac{100}{3} + \tfrac{75}{4}\right|$$

$$\therefore A = \tfrac{625}{24} \text{ square units}$$