THE LINE (Q 2, PAPER 2)

1999

2 (a) The point $(k, 1)$ lies on the line $4x-3y+15=0$.
Find the value of k.

(b) $p(4, 3)$, $q(-1, 0)$ and $r(10, 3)$ are three points.

(i) Find the slope of pq.

(ii) Find the equation of the line through r which is parallel to pq.

(iii) Find the equation of the line which is perpendicular to pq and which contains the origin.

(c) $a(0, 5)$, $b(x, 10)$ and $c(2x, x)$ are three points.
Find $|ab|$ in terms of x.
If $|ab| = |bc|$, calculate the two possible values of x.

SOLUTION

2 (a)

IS A POINT ON A LINE?
To show a point is on a line, put the point into the equation.

$(k, 1) \in 4x-3y+15=0$

$\Rightarrow 4(k)-3(1)+15=0$

$\Rightarrow 4k-3+15=0$

$\Rightarrow 4k+12=0$

$\Rightarrow 4k=-12$

$\therefore k=-3$

2 (b) (i)

$\boxed{m=\frac{y_2-y_1}{x_2-x_1}}$ 3

REMEMBER IT AS:

Slope $m=\frac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cc} p(4, 3) & q(-1, 0) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

Slope of pq: $m=\frac{0-3}{-1-4}=\frac{-3}{-5}=\frac{3}{5}$

2 (b) (ii)

Equation of line: Point $r(10, 3)$, slope $m=\frac{3}{5}$.

Parallel lines have the same slope.

$y-y_1=m(x-x_1)$

$\Rightarrow y-3=\frac{3}{5}(x-10)$

$\Rightarrow 5(y-3)=3(x-10)$

$\Rightarrow 5y-15=3x-30$

$\therefore 3x-5y-15=0$

2 (b) (iii)

FINDING THE PERPENDICULAR SLOPE: Invert the slope and change its sign.

Equation of line: Point (0, 0), slope $m=-\frac{5}{3}$.

$y-y_1=m(x-x_1)$

$\Rightarrow y-0=-\frac{5}{3}(x-0)$

$\Rightarrow 3y=-5x$

$\therefore 5x+3y=0$

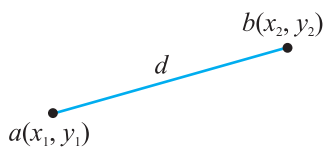
2 (c)

$\boxed{d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$ 1

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$d=\sqrt{(\text{Difference in } x\text{'s})^2+(\text{Difference in } y\text{'s})^2}$



$$\begin{array}{cc} a(0,\ 5) & b(x,\ 10) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$|ab|=\sqrt{(10-5)^2+(x-0)^2}$

$\Rightarrow |ab|=\sqrt{5^2+x^2}$

$\therefore |ab|=\sqrt{x^2+25}$

$$\begin{array}{cc} b(x,\ 10) & c(2x,\ x) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$|bc|=\sqrt{(2x-x)^2+(x-10)^2}$

$\therefore |bc|=\sqrt{x^2+(x-10)^2}$

$|ab|=|bc|$

$\Rightarrow \sqrt{x^2+25}=\sqrt{x^2+(x-10)^2}$ [Square both sides.]

$\Rightarrow x^2+25=x^2+(x-10)^2$ [Square out the bracket.]

$\Rightarrow 25=x^2-20x+100$

$\Rightarrow x^2-20x+75=0$ [Factorise the quadratic.]

$\Rightarrow (x-5)(x-15)=0$

$\therefore x=5,\ 15$