## The Line (Q 2, Paper 2)

## 1997

2 (a) Find the distance between the two points $(-5,1)$ and (7, -4).
(b) $L$ is the line $x-2 y+2=0$.
$M$ is the line $3 x+y-8=0$.
Find the co-ordinates of $p$, the point of intersection of $L$ and $M$.
$L$ and $M$ cut the $x$-axis at $q$ and $r$, respectively.
Find the area of triangle pqr.
(c) $K$ is the line which contains the points $a(0,4)$ and $b(3,0)$.

Find the equation of $K$.
$N$ is the line which is perpendicular to $K$ and which contains the origin.
Find the equation of $N$.
Investigate if $b$ is the image of $a$ under the axial symmetry in $N$.
Solution
2 (a)

$$
d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
$$

1
The distance between $a$ and $b$ is written as $|a b|$.
Remember the distance formula as:

$d=\sqrt{\left(\text { Difference in } x^{\prime} \mathrm{s}\right)^{2}+\left(\text { Difference in } y^{\prime} \mathrm{s}\right)^{2}}$

$$
\begin{array}{ccc}
\begin{array}{ccc}
(-5,1) & (7,-4) \\
\downarrow \downarrow & \downarrow & \downarrow \\
x_{1} y_{1} & x_{2} & y_{2}
\end{array} & d=\sqrt{(7-(-5))^{2}+(-4-1)^{2}} \\
& \Rightarrow d=\sqrt{(7+5)^{2}+(-4-1)^{2}} \\
& \Rightarrow d=\sqrt{(12)^{2}+(-5)^{2}}=\sqrt{144+25} \\
& & \therefore d=\sqrt{169}=13
\end{array}
$$

2 (b)

## Intersecting Lines

To find out where two lines intersect, solve their equations simultaneously.

$$
\begin{aligned}
& x-2 y+2=0 \ldots .(\mathbf{1}) \\
& 3 x+y-8=0 \ldots(2)(\times 2)
\end{aligned} \longrightarrow \left\lvert\, \begin{aligned}
& x-2 y+2=0 \\
& \frac{6 x+2 y-16}{}=0 \\
& 7 x-14=0 \Rightarrow 7 x=14 \Rightarrow x=2
\end{aligned}\right.
$$

Substitute this value of $x$ into Eqn. (1).
(2) $-2 y+2=0 \Rightarrow 4=2 y$
$\therefore y=2 \Rightarrow p(2,2)$ is the point of intersection.

$$
\text { To find the } x \text {-intercept: Put } y=0 \text {. }
$$ To find the $y$-intercept: Put $x=0$.

$L: x-2 y+2=0$
To find $x$-intercept, put $y=0: x-2(0)+2=0 \Rightarrow x=-2 \Rightarrow q(-2,0)$ is the $x$-intercept.
$M: 3 x+y-8=0$
To find $x$-intercept, put $y=0: 3 x+(0)-8=0 \Rightarrow 3 x=8 \Rightarrow x=\frac{8}{3} \Rightarrow r\left(\frac{8}{3}, 0\right)$ is the $x$-intercept.

The area, $A$, of $\Delta a o b$ with vertices
$o(0,0), a\left(x_{1}, y_{1}\right), b\left(x_{2}, y_{2}\right)$ is given by:
$A=\frac{1}{2}\left|x_{1} y_{2}-x_{2} y_{1}\right|$
6

## Steps

1. Translate one point to $(0,0)$.
2. Do the same translation to the other two points.
3. Apply the formula.

$$
\begin{array}{ll}
p(2,2) & \rightarrow(4,2) \\
q(-2,0) & \rightarrow(0,0) \\
r\left(\frac{8}{3}, 0\right) & \rightarrow\left(\frac{14}{3}, 0\right)
\end{array} \quad \begin{aligned}
& \text { Translate } q \text { to }(0,0) \text { by adding } 2 \text { to the } x \text { part and adding } \\
&
\end{aligned}
$$

| $(4,2)$ $\left(\frac{14}{3}, 0\right)$ <br> $\downarrow$  <br> $\downarrow$ $\downarrow$ <br> $x_{1} y_{1}$ $x_{2} y_{2}$ | $A=\frac{1}{2}\left\|(4)(0)-(2)\left(\frac{14}{3}\right)\right\|$$\quad$$\left.\Rightarrow A=\frac{1}{2}\left\|0-\frac{28}{3}\right\|=\frac{1}{2} 2 \frac{28}{3} \right\rvert\,$ <br> $\therefore A=\frac{14}{3}$ square units |
| :---: | :---: | :---: |

2 (c)

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \ldots \ldots . \text {. } 3
$$

## Remember it as:

$$
\text { Slope } m=\frac{\text { Difference in } y^{\prime} \mathrm{s}}{\text { Difference in } x^{\prime} \mathrm{s}}
$$

$$
\begin{array}{cc}
a(0,4) & b(3,0) \\
\downarrow \downarrow & \downarrow \downarrow \\
x_{1} y_{1} & x_{2} y_{2}
\end{array}
$$

$$
\text { Slope of } a b \text { : } m=\frac{0-4}{3-0}=\frac{-4}{3}=-\frac{4}{3}
$$

Equation of $K$ : Point $a(4,0)=\left(x_{1}, y_{1}\right)$, slope $m=-\frac{4}{3}$
The equation of a line is a formula satisfied by every point $(x, y)$ on the line.
Equation of a line: $y-y_{1}=m\left(x-x_{1}\right)$ 4
$y-y_{1}=m\left(x-x_{1}\right)$
$\Rightarrow y-4=-\frac{4}{3}(x-0)$
$\Rightarrow 3(y-4)=-4 x$
$\Rightarrow 3 y-12=-4 x$
$\Rightarrow 4 x+3 y-12=0$

Finding the perpendicular slope: Invert the slope and change its sign.
Equation of $N$ : Point $\left(x_{1}, y_{1}\right)=(0,0)$, slope $m=\frac{3}{4}$
$y-y_{1}=m\left(x-x_{1}\right)$
$\Rightarrow y-0=\frac{3}{4}(x-0)$
$\Rightarrow 4 y=3 x$
$\therefore 3 x-4 y=0$
To answer the last part plot the points and lines to give you a better idea of how to proceed.

Axial Symmetry: This is the movement of a point perpendicular to a line and out the same distance at right angles to the line.

If $b$ is the image of $a$ under an axial symmetry in $N$, then the point of intersection of $K$ and $N$ must be the same as the midpoint of [ab].


The formula for the midpoint, $c$, of the line segment [ab] is:

$$
\text { Midpoint }=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right) \ldots \ldots\left(2 a\left(x_{1}, y_{1}\right) \quad c(x, y)\right.
$$

Remember the midpoint formula as: Midpoint $=\left(\frac{\text { Add the } x^{\prime} \text { s }}{2}, \frac{\text { Add the } y \text { 's }}{2}\right)$

$$
\begin{array}{cc}
a(0,4) & b(3,0) \\
\downarrow \downarrow & \downarrow \downarrow \\
x_{1} y_{1} & x_{2} y_{2}
\end{array}
$$

$$
\text { Midpoint }=\left(\frac{0+3}{2}, \frac{4+0}{2}\right)=\left(\frac{3}{2}, \frac{4}{2}\right)=\left(\frac{3}{2}, 2\right)
$$

Find the point of intersection of $K$ and $N$

## Intersecting Lines

To find out where two lines intersect, solve their equations simultaneously.

$$
\begin{aligned}
4 x+3 y-12 & =0 \ldots(\mathbf{1})(\times 4) \\
3 x-4 y \quad & =0 \ldots(2)(\times 3)
\end{aligned} \longrightarrow \begin{aligned}
& 16 x+12 y-48=0 \\
& \frac{9 x-12 y}{}=0
\end{aligned}
$$

You can see the $x$ value of the point of intersection does not match the the $x$ value of the midpoint. Therefore, $b$ is not an image of $a$ under an axial symmetry in $N$.

