The Line (Q 2, Paper 2)

1996

2 (a) The line L contains the points $p(3, -1)$ and $q(0, 2)$.

(i) Find the slope of L.

(ii) Find the equation of L.

(iii) L intersects the x-axis at the point r. Find the coordinates of r.

(iv) Calculate the ratio

$$\frac{\text{area of triangle } rpo}{\text{area of triangle } pqo}$$

where o is the origin.

(b) The equation of the line M is $y - 4x - c = 0$.
M contains the point $p(1, 6)$.

(i) Find the value of c.

(ii) The origin is the midpoint of $[pq]$.
Find the equation of the line K if K is parallel to M and K contains the point q.

(iii) Find the equation of the line L if L is perpendicular to M and L contains the point q.

Solution

2 (a) (i)

$\boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$ **3**

Remember it as:

Slope $m = \dfrac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$

$$\begin{array}{cccc} p(3, & -1) & q(0, & 2) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of L: $m = \dfrac{2-(-1)}{0-3} = \dfrac{2+1}{0-3} = \dfrac{3}{-3} = -1$

2 (a) (ii)

The equation of a line is a formula satisfied by every point (x, y) on the line.

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$ **4**

Equation of L: point $q(0, 2) = (x_1, x_2)$, slope $m = -1$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = -1(x - 0)$

$\Rightarrow y - 2 = -x$

$\therefore x + y - 2 = 0$

2 (a) (iii)

To find the x-intercept: Put $y = 0$.
To find the y-intercept: Put $x = 0$.

To find the x-intercept of L, put $y = 0$.

$y=0: x+(0)-2=0 \Rightarrow x=2 \Rightarrow r(2,\ 0)$ is the x-intercept.

2 (a) (iv)

The area, A, of Δaob with vertices $o(0, 0)$, $a(x_1, y_1)$, $b(x_2, y_2)$ is given by:

$\boxed{y=0: x+(0)-}$ 6

Steps
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

Area of triangle rpo: One of the points is already (0, 0) so all you have to do is apply the formula to the other 2 points.

$$\begin{array}{cc} r(2,\ 0) & p(3,\ -1) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$A_1=\frac{1}{2}|(2)(-1)-(0)(3)|$

$\Rightarrow A_1=\frac{1}{2}|-2-0|=\frac{1}{2}|-2|=\frac{1}{2}(2)$

$\therefore A_1=1$ square unit

Area of triangle pqo:

$$\begin{array}{cc} p(3,\ -1) & q(0,\ 2) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$A_2=\frac{1}{2}|(3)(2)-(-1)(0)|$

$\Rightarrow A_2=\frac{1}{2}|6+0|=\frac{1}{2}|6|=\frac{1}{2}(6)$

$\therefore A_2=3$ square units

$$\frac{A_1}{A_2}=\frac{1}{3} \Rightarrow A_1 : A_2 = 1:3$$

2 (b) (i)

Is a point on a line?
To show a point is on a line, put the point into the equation.

$p(1,\ 6) \in y-4x-c=0$

$\Rightarrow (6)-4(1)-c=0$

$\Rightarrow 6-4-c=0$

$\Rightarrow 2-c=0$

$\therefore c=2$

2 (b) (ii)

To find the equation of K you need a point on K and the slope of K.

General form of a straight line
Every straight line can be written in the form: $ax + by + c = 0$.
You can read off the slope of a straight line from its equation.

Slope: $\boxed{m=-\left(\frac{a}{b}\right)}$ 5

Remember it as: Slope $m=-\left(\frac{\text{Number in front of } x}{\text{Number in front of } y}\right)$

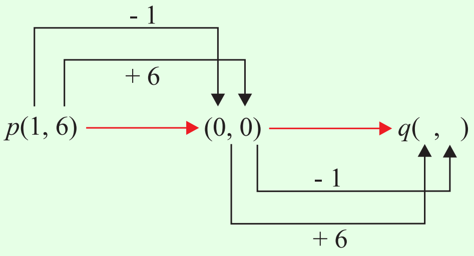
$M : y-4x-2=0 \Rightarrow 4x-y+2=0$

Slope of M: $m=-\frac{4}{-1}=4$

Parallel lines have the same slope.

Slope of K: $m = 4$

The image of $p(1, 6)$ through the origin $(0, 0)$ by a central symmetry is the point q.



$\therefore\ p(1,\ 6) \to (0,\ 0) \to q(-1,\ -6)$

Equation of K: Point $(x_1,\ y_1)=q(-1,\ -6)$, slope $m = 4$.

$y-y_1=m(x-x_1)$

$\Rightarrow y-(-6)=4(x-(-1))$

$\Rightarrow y+6=4(x+1)$

$\Rightarrow y+6=4x+4$

$\therefore 4x-y-2=0$

2 (b) (iii)

Finding the perpendicular slope: Invert the slope and change its sign.

Equation of L: Point $(x_1,\ y_1)=q(-1,\ -6)$, slope $m=-\frac{1}{4}$.

$y-y_1=m(x-x_1)$

$\Rightarrow y-(-6)=-\frac{1}{4}(x-(-1))$

$\Rightarrow 4(y+6)=-1(x+1)$

$\Rightarrow 4y+24=-x-1$

$\therefore x+4y+25=0$