DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

LESSON NO. 6: DIFFERENTIATION 4: CHAIN RULE

2006

7 (b) (ii) Given that $y=(5-x^2)^3$, find $\frac{dy}{dx}$ when $x = 2$.

SOLUTION

$$y=(u)^n \Rightarrow \frac{dy}{dx}=n(u)^{n-1}\times\frac{du}{dx} \quad \ldots 1$$

REMEMBER IT AS:

Push the power down in front of the bracket and subtract one from the power. Multiply by the differentiation of the inside of the bracket.

$$u=(5-x^2)\Rightarrow\frac{du}{dx}=0-2x=-2x$$

$$y=(5-x^2)^3\Rightarrow\frac{dy}{dx}=3(5-x^2)^2\times(-2x)$$

$$\Rightarrow\frac{dy}{dx}=-6x(5-x^2)^2 \quad \text{[Replace } x \text{ by 2.]}$$

$$\Rightarrow\left(\frac{dy}{dx}\right)_{x=2}=-6(2)(5-(2)^2)^2$$

$$=-12(5-4)^2=-12(1)^2=-12(1)=-12$$

2004

7 (b) (ii) Given that $y=(x^2-2x-3)^3$, show that $\frac{dy}{dx}=0$ when $x = 1$.

SOLUTION

$$y=(x^2-2x-3)^3$$

$$y=(u)^n \Rightarrow \frac{dy}{dx}=n(u)^{n-1}\times\frac{du}{dx} \quad \ldots 1$$

REMEMBER IT AS:

Push the power down in front of the bracket and subtract one from the power. Multiply by the differentiation of the inside of the bracket.

$$u=(x^2-2x-3)\Rightarrow\frac{du}{dx}=2x-2$$

$$y=(x^2-2x-3)^3$$

$$\Rightarrow\frac{dy}{dx}=3(x^2-2x-3)^2(2x-2)$$

$$\Rightarrow\frac{dy}{dx}=(6x-6)(x^2-2x-3)^2$$

2003

7 (b) (i) Differentiate $(3x^3-2x^2+2)^4$ with respect to x.

SOLUTION

$$y=(u)^n \Rightarrow \frac{dy}{dx}=n(u)^{n-1}\times\frac{du}{dx} \quad \ldots \; 1$$

REMEMBER IT AS:

Push the power down in front of the bracket and subtract one from the power. Multiply by the differentiation of the inside of the bracket.

$$u=(3x^3-2x^2+2)\Rightarrow\frac{du}{dx}=9x^2-4x$$

$$y=(3x^3-2x^2+2)^4\Rightarrow\frac{dy}{dx}=4(3x^3-2x^2+2)^3(9x^2-4x)=(36x^2-16x)(3x^3-2x^2+2)^3$$

2002

6 (b) (i) Find $\frac{dy}{dx}$ where $y=(x-1)^7$ and evaluate your answer at $x = 2$.

SOLUTION

$$y=(u)^n \Rightarrow \frac{dy}{dx}=n(u)^{n-1}\times\frac{du}{dx} \quad \ldots \; 1$$

REMEMBER IT AS:

Push the power down in front of the bracket and subtract one from the power. Multiply by the differentiation of the inside of the bracket.

$$y=(x-1)^7$$

$$\Rightarrow\frac{dy}{dx}=7(x-1)^6(1)=7(x-1)^6$$

$$u=(x-1)\Rightarrow\frac{du}{dx}=1$$

2001

7 (b) (ii) Find the value of $\frac{dy}{dx}$ at $x = 0$ when $y=(x^2-7x+1)^5$.

SOLUTION

$$u=x^2-7x+1\Rightarrow\frac{du}{dx}=2x-7$$

$$y=(x^2-7x+1)^5\Rightarrow\frac{dy}{dx}=5(x^2-7x+1)^4(2x-7)$$

$$\Rightarrow\frac{dy}{dx}=(10x-35)(x^2-7x+1)^4$$

$$\Rightarrow\left(\frac{dy}{dx}\right)_{x=0}=(10(0)-35)((0)^2-7(0)+1)^4=(-35)(1)^4=-35$$

2000

7 (b) (ii) Find $\frac{dy}{dx}$ when $y=(x^2+5x-1)^3$.

SOLUTION

$$\boxed{y=(u)^n \Rightarrow \frac{dy}{dx}=n(u)^{n-1}\times\frac{du}{dx}} \quad \ldots \text{ 1}$$

REMEMBER IT AS:

Push the power down in front of the bracket and subtract one from the power. Multiply by the differentiation of the inside of the bracket.

$$u=(x^2+5x-1) \Rightarrow \frac{du}{dx}=2x+5$$

$y=(x^2+5x-1)^3$

$\Rightarrow \frac{dy}{dx}=3(x^2+5x-1)^2(2x+5)$

$\Rightarrow \frac{dy}{dx}=(6x+15)(x^2+5x-1)^2$

1999

7 (b) (i) Find $\frac{dy}{dx}$ when $y=(3-7x)^5$.

SOLUTION

$$\boxed{y=(u)^n \Rightarrow \frac{dy}{dx}=n(u)^{n-1}\times\frac{du}{dx}} \quad \ldots \text{ 1}$$

REMEMBER IT AS:

Push the power down in front of the bracket and subtract one from the power. Multiply by the differentiation of the inside of the bracket.

$$u=3-7x \Rightarrow \frac{du}{dx}=-7$$

$y=(3-7x)^5 \Rightarrow \frac{dy}{dx}=5(3-7x)^4(-7)=-35(3-7x)^4$

1998

7 (b) (ii) Find $\frac{dy}{dx}$ when $y=(4-3x^2)^7$ and write down the range of values of x for which $\frac{dy}{dx}>0$.

SOLUTION

$$\boxed{y=(u)^n \Rightarrow \frac{dy}{dx}=n(u)^{n-1}\times\frac{du}{dx}} \quad \dots \text{ 1}$$

REMEMBER IT AS:

Push the power down in front of the bracket and subtract one from the power. Multiply by the differentiation of the inside of the bracket.

$$\boxed{u=(4-3x^2)\Rightarrow\frac{du}{dx}=-6x}$$

$$y=(4-3x^2)^7 \Rightarrow \frac{dy}{dx}=7(4-3x^2)^6(-6x)=-42x(4-3x^2)^6$$

$$\frac{dy}{dx}>0 \Rightarrow -42x(4-3x^2)^6>0$$

What values of x make this statement true?

$(4-3x^2)^6>0$ is true for all values of x. No matter what value is inside the bracket, once you raise it to an even power the value will be positive.

This means that x must be negative so that the $-42x$ is also positive.
Therefore, $x<0$.

1997

7 (b) (ii) Find the value of $\frac{dy}{dx}$ at $x=-1$ when $y=(3x+1)^4$.

SOLUTION

$$\boxed{y=(u)^n \Rightarrow \frac{dy}{dx}=n(u)^{n-1}\times\frac{du}{dx}} \quad \dots \text{ 1}$$

REMEMBER IT AS:

Push the power down in front of the bracket and subtract one from the power. Multiply by the differentiation of the inside of the bracket.

$$\boxed{u=3x+1\Rightarrow\frac{du}{dx}=3}$$

$$y=(3x+1)^4 \Rightarrow \frac{dy}{dx}=4(3x+1)^3(3)=12(3x+1)^3$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=-1}=12(3(-1)+1)^3=12(-3+1)^3$$

$$=12(-2)^3=12(-8)=-96$$

1996

7 (b) (ii) Differentiate $\left(x^5 - \frac{1}{x^2}\right)^7$ with respect to x, $x \neq 0$.

SOLUTION

$$\boxed{y = (u)^n \Rightarrow \frac{dy}{dx} = n(u)^{n-1} \times \frac{du}{dx}} \quad \ldots \text{1}$$

REMEMBER IT AS:

Push the power down in front of the bracket and subtract one from the power. Multiply by the differentiation of the inside of the bracket.

$$u = x^5 - \frac{1}{x^2} = x^5 - x^{-2} \Rightarrow \frac{du}{dx} = 5x^4 + 2x^{-3} = 5x^4 + \frac{2}{x^3}$$

$$y = \left(x^5 - \frac{1}{x^2}\right)^7 \Rightarrow \frac{dy}{dx} = 7\left(x^5 - \frac{1}{x^2}\right)^6\left(5x^4 + \frac{2}{x^3}\right)$$

$$\therefore \frac{dy}{dx} = \left(35x^4 + \frac{14}{x^3}\right)\left(x^5 - \frac{1}{x^2}\right)^6$$