DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

LESSON NO. 5: DIFFERENTIATION 3: QUOTIENTS

2007

7 (b) (ii) Given that $y = \frac{3x}{2x+3}$, find $\frac{dy}{dx}$.

Write your answer in the form $\frac{k}{(2x+3)^n}$, where $k, n \in \mathbf{N}$.

SOLUTION

THE QUOTIENT RULE: If $y = \frac{u}{v}$ then:

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \text{.......} \ \mathbf{3}$$

$u = 3x \Rightarrow \frac{du}{dx} = 3$

$v = 2x+3 \Rightarrow \frac{dv}{dx} = 2+0 = 2$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(2x+3)3-(3x)2}{(2x+3)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{6x+9-6x}{(2x+3)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{9}{(2x+3)^2}$$

2006

7 (b) (i) Differentiate $\frac{x^2-1}{x+1}$ with respect to x and write your answer in its simplest form.

SOLUTION

THE QUOTIENT RULE: If $y = \frac{u}{v}$ then:

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \text{.......} \ \mathbf{3}$$

$u = x^2 - 1 \Rightarrow \frac{du}{dx} = 2x - 0 = 2x$

$v = x+1 \Rightarrow \frac{dv}{dx} = 1+0 = 1$

$$y = \frac{x^2-1}{x+1} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(x+1)(2x)-(x^2-1)(1)}{(x+1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2+2x-x^2+1}{(x+1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+2x+1}{(x+1)^2}$$

[You can factorise the top: $x^2+2x+1 = (x+1)(x+1) = (x+1)^2$.]

$$\Rightarrow \frac{dy}{dx} = \frac{(x+1)^2}{(x+1)^2} = 1$$

2005

7 (b) (ii) Given that $y=\frac{x^2}{x-1}$, find $\frac{dy}{dx}$ when $x = 3$.

SOLUTION

$$y=\frac{x^2}{x-1}$$

$$u=x^2 \Rightarrow \frac{du}{dx}=2x$$
$$v=x-1 \Rightarrow \frac{dv}{dx}=1$$

THE QUOTIENT RULE: If $y=\frac{u}{v}$ then:

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2} \quad \text{.......} \; 3$$

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}=\frac{(x-1)(2x)-x^2(1)}{(x-1)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2x^2-2x-x^2}{(x-1)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{x^2-2x}{(x-1)^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=3}=\frac{(3)^2-2(3)}{((3)-1)^2}=\frac{9-6}{(2)^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=3}=\frac{3}{4}$$

2002

7 (b) (ii) Differentiate $\frac{2x}{x-1}$ with respect to x and simplify your answer.

SOLUTION

$$y=\frac{2x}{x-1} \Rightarrow \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}=\frac{(x-1)2-2x(1)}{(x-1)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2x-2-2x}{(x-1)^2}=-\frac{2}{(x-1)^2}$$

THE QUOTIENT RULE: If $y=\frac{u}{v}$ then:

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2} \quad \text{.......} \; 3$$

$$u=2x \Rightarrow \frac{du}{dx}=2$$
$$v=(x-1) \Rightarrow \frac{dv}{dx}=1$$

2001

7 (b) (i) Find $\frac{dy}{dx}$ when $y=\frac{x^2}{x-4}$, $x \neq 4$.

SOLUTION

THE QUOTIENT RULE: If $y=\frac{u}{v}$ then:

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2} \quad .... \; ❸$$

$$u=x^2 \Rightarrow \frac{du}{dx}=2x$$

$$v=(x-4) \Rightarrow \frac{du}{dx}=1$$

$$y=\frac{x^2}{(x-4)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}=\frac{(x-4)(2x)-x^2(1)}{(x-1)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2x^2-8x-x^2}{(x-1)^2}=\frac{x^2-8x}{(x-1)^2}$$

2000

7 (b) (i) Find $\frac{dy}{dx}$ when $y=\frac{2x-7}{x-1}$, $x \neq 1$.

SOLUTION

THE QUOTIENT RULE: If $y=\frac{u}{v}$ then:

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2} \quad .... \; ❸$$

$$u=(2x-7) \Rightarrow \frac{du}{dx}=2$$

$$v=(x-1) \Rightarrow \frac{dv}{dx}=1$$

$$y=\frac{2x-7}{x-1}$$

$$\Rightarrow \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}=\frac{(x-1)2-(2x-7)1}{(x-1)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2x-2-2x+7}{(x-1)^2}=\frac{5}{(x-1)^2}$$

1999

7 (b) (ii) Find $\frac{dy}{dx}$ when $y = \frac{x^2}{1-x}$, $x \neq 1$. Show that $\frac{dy}{dx} = 0$ at $x = 0$.

SOLUTION

$$u = x^2 \Rightarrow \frac{du}{dx} = 2x$$

$$v = (1-x) \Rightarrow \frac{dv}{dx} = -1$$

THE QUOTIENT RULE: If $y = \frac{u}{v}$ then:

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \text{.......} \; ❸$$

$$y = \frac{x^2}{1-x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(1-x)(2x) - x^2(-1)}{(1-x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x - 2x^2 + x^2}{(1-x)^2} = \frac{2x - x^2}{(1-x)^2}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \frac{2(0) - (0)^2}{(1-0)^2} = \frac{0}{1} = 0$$

1998

7 (b) (i) Find $\frac{dy}{dx}$ when $y = \frac{2x}{x^2+1}$.

SOLUTION

$$u = 2x \Rightarrow \frac{du}{dx} = 2$$

$$v = (x^2+1) \Rightarrow \frac{dv}{dx} = 2x$$

THE QUOTIENT RULE: If $y = \frac{u}{v}$ then:

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \text{.......} \; ❸$$

$$y = \frac{2x}{x^2+1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(x^2+1)2 - 2x(2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + 2 - 4x^2}{(x^2+1)^2} = \frac{2 - 2x^2}{(x^2+1)^2}$$

1996

7 (b) (i) Find $\frac{dy}{dx}$ when $y = \frac{2x}{4-x^2}$, for $x \in \mathbf{R}$ and $x \neq \pm 2$.

Show that $\frac{dy}{dx} > 0$.

SOLUTION

$u = 2x \Rightarrow \frac{du}{dx} = 2$

$v = 4 - x^2 \Rightarrow \frac{dv}{dx} = -2x$

THE QUOTIENT RULE: If $y = \frac{u}{v}$ then:

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \text{.......} \; ❸$$

$$y = \frac{2x}{4-x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(4-x^2)(2) - 2x(-2x)}{(4-x^2)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{8 - 2x^2 + 4x^2}{(4-x^2)^2} = \frac{2x^2+8}{(4-x^2)^2}$$

$\frac{dy}{dx} = \frac{2x^2+8}{(4-x^2)^2} > 0$ for both positive and negative values of x as when they are squared the answer will be positive.