DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

LESSON NO. 4: DIFFERENTIATION 2: PRODUCTS

2007

7 (b) (i) Differentiate $(x^2+9)(4x^3+5)$ with respect to x.

SOLUTION

THE PRODUCT RULE: If $y = u \times v$ then:

$$\boxed{\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}} \text{.......} \; \textbf{2}$$

$$u = x^2+9 \Rightarrow \frac{du}{dx} = 2x + 0 = 2x$$

$$v = 4x^3+5 \Rightarrow \frac{dv}{dx} = 4\times 3x^2 + 0 = 12x^2$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = (x^2+9)(12x^2) + (4x^3+5)(2x)$$

$$\Rightarrow \frac{dy}{dx} = 12x^4 + 108x^2 + 8x^4 + 10x$$

$$\Rightarrow \frac{dy}{dx} = 20x^4 + 108x^2 + 10x$$

2005

7 (b) (i) Differentiate $(3x^2-2)(x^2+4)$ with respect to x.

SOLUTION

THE PRODUCT RULE: If $y = u \times v$ then:

$$\boxed{\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}} \text{.......} \; \textbf{2}$$

$$y = (3x^2-2)(x^2+4)$$

$$u = (3x^2-2) \Rightarrow \frac{du}{dx} = 3\times 2x - 0 = 6x$$

$$v = (x^2+4) \Rightarrow \frac{dv}{dx} = 2x + 0 = 2x$$

$$\therefore \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{dv}{dx} = (3x^2-2)(2x) + (x^2+4)(6x)$$

$$\Rightarrow \frac{dy}{dx} = 6x^3 - 4x + 6x^3 + 24x = 12x^3 + 20x$$

2004

7 (b) (i) Differentiate $(x^2-4)(x^2+3x)$ with respect to x.

SOLUTION

$y=(x^2-4)(x^2+3x)$

THE PRODUCT RULE: If $y=u\times v$ then:

$$\boxed{\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}} \quad \text{.......} \; 2$$

$$u=(x^2-4)\Rightarrow\frac{du}{dx}=2x-0=2x$$

$$v=(x^2+3x)\Rightarrow\frac{dv}{dx}=2x+3$$

$$\therefore\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}=(x^2-4)(2x+3)+(x^2+3x)(2x)$$

$$\Rightarrow\frac{dy}{dx}=2x^3+3x^2-8x-12+2x^3+6x^2$$

$$\Rightarrow\frac{dy}{dx}=4x^3+9x^2-8x-12$$

2003

7 (b) (ii) Given that $y=(5x^2+3)(4-x^2)$, find $\frac{dy}{dx}$ when $x=1$.

SOLUTION

$y=(5x^2+3)(4-x^2)$

$$\Rightarrow\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}=(5x^2+3)(-2x)+(4-x^2)(10x)$$

$$\Rightarrow\frac{dy}{dx}=-10x^3-6x+40x-10x^3$$

$$\Rightarrow\frac{dy}{dx}=-20x^3+34x$$

$$\therefore\left(\frac{dy}{dx}\right)_{x=1}=-20(1)^3+34(1)=-20+34=14$$

THE PRODUCT RULE: If $y=u\times v$ then:

$$\boxed{\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}} \quad \text{.......} \; 2$$

$$u=(5x^2+3)\Rightarrow\frac{du}{dx}=10x$$

$$v=(4-x^2)\Rightarrow\frac{dv}{dx}=-2x$$

2002

6 (b) (ii) Find $\frac{dy}{dx}$ where $y=(x^3-3)(x^2-4)$ and simplify your answer.

SOLUTION

$y=(x^3-3)(x^2-4)$

$$\Rightarrow\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}=(x^3-3)(2x)+(x^2-4)(3x^2)$$

$$\Rightarrow\frac{dy}{dx}=2x^4-6x+3x^4-12x^2=5x^4-12x^2-6x$$

THE PRODUCT RULE: If $y=u\times v$ then:

$$\boxed{\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}} \quad \text{.......} \; 2$$

$$u=(x^3-3)\Rightarrow\frac{du}{dx}=3x^2$$

$$v=(x^2-4)\Rightarrow\frac{dv}{dx}=2x$$

1997

7 (b) (i) Find $\frac{dy}{dx}$ when $y=(x^2-3)(1-x)$.

Solution

You can multiply out the brackets and differentiate term by term or you can use the product rule.

Method 1: Multiply out the brackets.

$$y=(x^2-3)(1-x)=x^2-x^3-3+3x$$

$$\therefore y=-x^3+x^2+3x-3$$

$$\Rightarrow \frac{dy}{dx}=-3x^2+2x+3$$

Method 2: Product rule

$$u=(x^2-3)\Rightarrow \frac{du}{dx}=2x$$

$$v=(1-x)\Rightarrow \frac{dv}{dx}=-1$$

The Product Rule: If $y=u\times v$ then:

$$\boxed{\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}} \;\ldots\ldots\; 2$$

$$y=(x^2-3)(1-x)$$

$$\therefore \frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}=(x^2-3)(-1)+(1-x)(2x)$$

$$\Rightarrow \frac{dy}{dx}=-x^2+3+2x-2x^2$$

$$\Rightarrow \frac{dy}{dx}=-3x^2+2x+3$$