Differentiation & Functions (Q 6, 7 & 8, Paper 1)

Lesson No. 3: Differentiation 1: Sums of Terms

2007

6 (a) Let $g(x) = x^2 - 6x,\ x \in \mathbf{R}$.

(i) Write down $g'(x)$, the derivative of $g(x)$.

(ii) For what value of x is $g'(x) = 0$?

7 (a) Differentiate $6x^4 - 3x^2 + 7x$ with respect to x.

Solution

6 (a) (i)

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \ldots\ldots \textbf{1}$$

Remember it as:

Multiply down by the power and subtract one from the power.

Multiply by a constant rule: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$$g(x) = x^2 - 6x \Rightarrow g'(x) = 2x - 6$$

6 (a) (ii)

$$g'(x) = 0 \Rightarrow 2x - 6 = 0$$

$$\Rightarrow 2x = 6 \Rightarrow x = 3$$

7 (a)

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \ldots\ldots \textbf{1}$$

Remember it as:

Multiply down by the power and subtract one from the power.

Multiply by a constant rule: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$$y = 6x^4 - 3x^2 + 7x \Rightarrow \frac{dy}{dx} = 24x^3 - 6x + 7$$

2006

7 (a) Differentiate $5x^3 - 4x + 7$ with respect to x.

Solution

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \ldots\ldots \textbf{1}$$

Remember it as:

Multiply down by the power and subtract one from the power.

Constant Rule: If y = Constant $\Rightarrow \frac{dy}{dx} = 0$

Multiply by a constant rule: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$$y = 5x^3 - 4x + 7$$

$$\Rightarrow \frac{dy}{dx} = 5 \times 3x^2 - 4 \times 1 + 0$$

$$\Rightarrow \frac{dy}{dx} = 15x^2 - 4$$

2005

7 (a) Differentiate $9+3x-5x^2$ with respect to x.

SOLUTION

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \text{.....} \ (1)$$

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \frac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$$y = 9+3x-5x^2$$

$$\Rightarrow \frac{dy}{dx} = 0+3\times 1-5\times 2x$$

$$\Rightarrow \frac{dy}{dx} = 3-10x$$

2004

7 (a) Differentiate with respect to x:

(i) $2x^5$

(ii) $4(3-x^2)$.

SOLUTION

7 (a) (i)

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \text{.....} \ (1)$$

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \frac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$$y = 2x^5 \Rightarrow \frac{dy}{dx} = 2\times 5x^4 = 10x^4$$

7 (a) (ii)

$$y = 4(3-x^2) = 12-4x^2$$

$$\Rightarrow \frac{dy}{dx} = 0-4\times 2x = -8x$$

2003

7 (a) Differentiate with respect to x:

(i) x^3

(ii) $\dfrac{x^2 - x^4}{2}$.

SOLUTION

7 (a) (i)

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \ldots\ldots \textbf{1}$$

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \dfrac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\dfrac{dy}{dx} = c \times \dfrac{du}{dx}$.

$$y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2$$

7 (a) (ii)

$$y = \frac{x^2 - x^4}{2} = \tfrac{1}{2}x^2 - \tfrac{1}{2}x^4$$

$$\Rightarrow \frac{dy}{dx} = \tfrac{1}{2} \times 2x - \tfrac{1}{2} \times 4x^3 = x - 2x^3$$

2002

7 (a) Differentiate $7x^3 - 3x^2 + 9x$ with respect to x.

(b) (i) Differentiate $x^5 - 17 + \dfrac{1}{x^5}$ with respect to x.

SOLUTION

7 (a)

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \ldots\ldots \textbf{1}$$

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \dfrac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\dfrac{dy}{dx} = c \times \dfrac{du}{dx}$.

$$y = 7x^3 - 3x^2 + 9x$$

$$\Rightarrow \frac{dy}{dx} = 7 \times 3x^2 - 3 \times 2x + 9 = 21x^2 - 6x + 9$$

7 (b) (i)

$$y = x^5 - 17 + \frac{1}{x^5} = x^5 - 17 + x^{-5}$$

$$\Rightarrow \frac{dy}{dx} = 5x^4 - 0 - 5x^{-6}$$

$$\Rightarrow \frac{dy}{dx} = 5x^4 - \frac{5}{x^6}$$

POWER RULES

4. $a^{-n} = \dfrac{1}{a^n}$ **Ex.** $x^{-3} = \dfrac{1}{x^3}$

2001

7 (a) Differentiate with respect to x

(i) $6x^5 + x^2$

(ii) $(x-3)(x+3)$

8 (a) Let $g(x) = x^4 - 32x$ for $x \in \mathbf{R}$.

(i) Write down $g'(x)$, the derivative of $g(x)$.

(ii) For what value of x is $g'(x) = 0$?

SOLUTION

7 (a) (i)

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \text{.....} \ (1)$$

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \frac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$$y = 6x^5 + x^2$$

$$\Rightarrow \frac{dy}{dx} = 6 \times 5x^4 + 2x = 30x^4 + 2x$$

7 (a) (ii)

You could use the product rule but it is easier to multiply out the brackets and differentiate term by term.

$$y = (x-3)(x+3) = x^2 + 3x - 3x - 9 = x^2 - 9$$

$$\Rightarrow \frac{dy}{dx} = 2x - 0 = 2x$$

8 (a) (i)

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \text{.....} \ (1)$$

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \frac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$$g(x) = x^4 - 32x$$

$$\Rightarrow g'(x) = 4x^3 - 32$$

8 (a) (ii)

$$g'(x) = 0 \Rightarrow 4x^3 - 32 = 0$$

$$\Rightarrow 4x^3 = 32$$

$$\Rightarrow x^3 = 8$$

$$\therefore x = 2$$

2000

7 (a) Differentiate with respect to x

(i) $4x^2+5$

(ii) $9x-x^3$.

SOLUTION

7 (a) (i)

$$y=x^n \Rightarrow \frac{dy}{dx}=nx^{n-1} \quad \ldots\ldots \textbf{1}$$

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \frac{dy}{dx}=0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx}=c\times\frac{du}{dx}$.

$$y=4x^2+5$$

$$\Rightarrow \frac{dy}{dx}=4\times 2x+0=8x$$

7 (a) (ii)

$$y=9x-x^3$$

$$\Rightarrow \frac{dy}{dx}=9-3x^2$$

1999

7 (a) Differentiate

$$2x^3-7$$

with respect to x.

SOLUTION

$$y=x^n \Rightarrow \frac{dy}{dx}=nx^{n-1} \quad \ldots\ldots \textbf{1}$$

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \frac{dy}{dx}=0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx}=c\times\frac{du}{dx}$.

$$y=2x^3-7$$

$$\Rightarrow \frac{dy}{dx}=2\times 3x^2-0=6x^2$$

1998

7 (a) Differentiate with respect to x

(i) $x^2 - 3x$

(ii) $\dfrac{1}{x^2}$.

SOLUTION

7 (a) (i)

$y = x^n \Rightarrow \dfrac{dy}{dx} = nx^{n-1}$ **1**

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \dfrac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\dfrac{dy}{dx} = c \times \dfrac{du}{dx}$.

$y = x^2 - 3x$

$\Rightarrow \dfrac{dy}{dx} = 2x - 3$

7 (a) (ii)

$y = \dfrac{1}{x^2} = x^{-2}$

$\Rightarrow \dfrac{dy}{dx} = -2x^{-3} = -\dfrac{2}{x^3}$

POWER RULES

4. $a^{-n} = \dfrac{1}{a^n}$ **Ex.** $x^{-3} = \dfrac{1}{x^3}$

1997

7 (a) Differentiate with respect to x

(i) $-x^2$

(ii) $x^4 + x + 1$.

8 (a) Let $f(x) = x^2 - 4x$, for $x \in \mathbf{R}$.

Find $f'(x)$, the derivative of $f(x)$.

For what value of x is $f'(x) = 0$?

SOLUTION

7 (a) (i)

$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$ **1**

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \frac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$$y = -x^2 \Rightarrow \frac{dy}{dx} = -1 \times 2x = -2x$$

7 (a) (ii)

$$y = x^4 + x + 1 \Rightarrow \frac{dy}{dx} = 4x^3 + 1 + 0 = 4x^3 + 1$$

8 (a)

$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$ **1**

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \frac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$y = f(x) = x^2 - 4x$

$\Rightarrow \frac{dy}{dx} = f'(x) = 2x - 4$

$f'(x) = 0 \Rightarrow 2x - 4 = 0$

$\Rightarrow 2x = 4$

$\therefore x = 2$

1996

8 (a) Find $\frac{ds}{dt}$ when $s = 6t^2 - 3t + 7$.

SOLUTION

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \text{.....} \ (1)$$

REMEMBER IT AS:

Multiply down by the power and subtract one from the power.

CONSTANT RULE: If y = Constant $\Rightarrow \frac{dy}{dx} = 0$

MULTIPLY BY A CONSTANT RULE: If $y = cu$, where c is a constant and u is a function of x, $\frac{dy}{dx} = c \times \frac{du}{dx}$.

$s = 6t^2 - 3t + 7$

$\Rightarrow \frac{ds}{dt} = 6 \times 2t - 3 + 0 = 12t - 3$