# Differentiation & Functions (Q 6, 7 & 8, Paper 1)

## Lesson No. 1: Working with Functions

**2007**

8 (a) Let $f(x)=\frac{1}{4}(6-2x)$ for $x \in \mathbf{R}$. Evaluate $f(5)$.

**Solution**

$f(x)=\frac{1}{4}(6-2x)$

$\Rightarrow f(5)=\frac{1}{4}(6-2(5))$

$\Rightarrow f(5)=\frac{1}{4}(6-10)$

$\Rightarrow f(5)=\frac{1}{4}(-4)$

$\Rightarrow f(5)=-1$

**2006**

8 (a) Let $g(x)=\dfrac{3}{x+1}$, $x \in \mathbf{R}$, $x \neq -1$.

Evaluate $g(0.5)-g(-0.5)$.

(b) Let $h(x)=x^2+2x-1$, $x \in \mathbf{R}$.

(i) Simplify $h(x-5)$.

(ii) Find the value of $x$ for which $h(x-5)=h(x)-5$.

**Solution**

**8 (a)**

$g(x)=\dfrac{3}{x+1}$

$\therefore g(0.5)=\dfrac{3}{0.5+1}=\dfrac{3}{1.5}=2$

$\therefore g(-0.5)=\dfrac{3}{-0.5+1}=\dfrac{3}{0.5}=6$

$\Rightarrow g(0.5)-g(-0.5)=2-6=-4$

**8 (b) (i)**

$h(x)=x^2+2x-1$ [Replace $x$ by $(x-5)$.]

$\Rightarrow h(x-5)=(x-5)^2+2(x-5)-1$

$=x^2-10x+25+2x-10-1$

$=x^2-8x+14$

**8 (b) (ii)**

$h(x-5)=h(x)-5$

$\Rightarrow x^2-8x+14=x^2+2x-1-5$

$\Rightarrow -8x+14=2x-6$

$\Rightarrow 14+6=2x+8x$

$\Rightarrow 20=10x$

$\therefore x=2$

**2005**

6 (a) Let $g(x) = \frac{x+5}{2}$, $x \in \mathbf{R}$.

Find $g(0) + g(2)$.

**SOLUTION**

**6 (a) (i)**

Period = 8

Range = [−1, 2]

Every periodic function has two important features:
**1. PERIOD:**
The length of the wave along the *x*-axis before it repeats itself.
**2. RANGE:**
This is the interval between the lowest *y* value and the highest *y* value.

**6 (a) (ii)**

$f(44) = f(4) = 2$

The value of the function at any value of *x* can be worked out from the first wave by dividing the value of *x* by the period and finding the remainder.

$f(x) = f(\text{Remainder})$

**2004**

6 (a) Let $g(x) = 1 - kx$.

Given that $g(-3) = 13$, find the value of $k$.

8 (a) Let $g(x) = 3x - 7$.

(i) Find $g(7)$.

(ii) Find the value of $k$ for which $g(7) = k[g(0)]$.

**SOLUTION**

**6 (a)**

$g(x) = 1 - kx$

$g(-3) = 13 \Rightarrow 1 - k(-3) = 13$

$\Rightarrow 1 + 3k = 13$

$\Rightarrow 3k = 12$

$\therefore k = 4$

**8 (a) (i)**

$g(x) = 3x - 7$

$\Rightarrow g(7) = 3(7) - 7 = 21 - 7 = 14$

**8 (a) (ii)**

$g(7) = k[g(0)]$

$\Rightarrow 14 = k[3(0) - 7]$

$\Rightarrow 14 = k[-7]$

$\therefore k = -2$

**2003**

6 (a) Let $g(x) = \frac{2x}{3} - 1.$

Find the value of $x$ for which $g(x) = 5$.

8 (b) (i) The function $g$ is defined for natural numbers by the rule:

0 if is even.

1 if is odd

Find $g(13) + g(14) + g(15)$.

(ii) Given that $h(x) = x^2$, write down $h(x + 3)$.

Hence, find the value of $x$ for which $h(x) = h(x + 3)$.

**Solution**

**6 (a)**

$g(x) = 5$

$\Rightarrow \frac{2x}{3} - 1 = 5$

$\Rightarrow \frac{2x}{3} = 6$ [Multiply across by 3.]

$\Rightarrow 2x = 18$

$\Rightarrow x = 9$

**8 (b) (i)**

$g(13) = 1$ (because 13 is an odd number)

$g(14) = 0$ (because 14 is an even number)

$g(15) = 1$ (because 15 is an odd number)

$\therefore g(13) + g(14) + g(15) = 0 + 1 + 0 = 1$

**8 (b) (ii)**

$h(x) = x^2$

$\therefore h(x+3) = (x+3)^2 = x^2 + 6x + 9$

$h(x) = h(x+3)$

$\therefore x^2 = x^2 + 6x + 9 \Rightarrow 0 = 6x + 9$

$\Rightarrow -9 = 6x \Rightarrow -\frac{9}{6} = x$

$\therefore x = -\frac{3}{2}$

**2002**

6 (a) Let $f(x)=\frac{1}{3}(x-8)$ for $x \in \mathbf{R}$.

Evaluate $f(5)$.

**SOLUTION**

$f(x)=\frac{1}{3}(x-8)$

$\Rightarrow f(5)=\frac{1}{3}(5-8)=\frac{1}{3}(-3)=-1$

**2001**

6 (a) Let $g(x)=\dfrac{1}{x^2+1}$ for $x \in \mathbf{R}$.

Evaluate

(i) $g(2)$

(ii) $g(3)$ and write your answers as decimals.

**SOLUTION**

$g(x)=\dfrac{1}{x^2+1}$

$\Rightarrow g(2)=\dfrac{1}{(2)^2+1}=\dfrac{1}{4+1}=\dfrac{1}{5}=0.2$

$\Rightarrow g(3)=\dfrac{1}{(3)^2+1}=\dfrac{1}{9+1}=\dfrac{1}{10}=0.1$

**2000**

8 (a) Let $p(x)=3x-12$.

For what values of $x$ is $p(x) < 0$ where $x$ is a positive whole number?

**SOLUTION**

$p(x)=3x-12$

$\therefore 3x-12<0$

$\Rightarrow 3x<12$

$\Rightarrow x<4$ [The whole positive numbers less than 4 are 1, 2 and 3.]

$\therefore x=\{1, 2, 3\}$

**1999**

6 (a) Let $f(x)=2(3x-1),\ x\in\mathbf{R}$.
Find the value of $x$ for which $f(x) = 0$.

**SOLUTION**

$f(x)=2(3x-1)$

$f(x)=0\Rightarrow 2(3x-1)=0$

$\Rightarrow 3x-1=0$

$\Rightarrow 3x=1$

$\therefore x=\frac{1}{3}$

**1998**

6 (a) If $f(x)=5x-8$ and $g(x)=13-2x$, find the value of $x$ for which
$f(x) = g(x)$.

**SOLUTION**

$f(x)=g(x)$

$\Rightarrow 5x-8=13-2x$

$\Rightarrow 5x+2x=13+8$

$\Rightarrow 7x=21$

$\therefore x=3$

**1996**

6 (a) Let $f(x) = 3x + k,\ x\in\mathbf{R}$.
If $f(5) = 0$, find the value of $k$.

**SOLUTION**

$f(x)=3x+k$

$f(5)=0\Rightarrow 3(5)+k=0$

$\Rightarrow 15+k=0$

$\therefore k=-15$