# COMPLEX NUMBERS (Q 4, PAPER 1)

## LESSON NO. 7: EQUATIONS II

**2006**

4 (b) (i) Solve $z^2 - 4z + 29 = 0$.

Write your answers in the form $x + yi$ where $x, y \in \mathbf{R}$.

**SOLUTION**

$z^2 - 4z + 29 = 0$

$\therefore z = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(29)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{16 - 116}}{2} = \dfrac{4 \pm \sqrt{-100}}{2}$

$= \dfrac{4 \pm 10i}{2} = 2 \pm 5i$

$z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ....... **3**

$a = 1$
$b = -4$
$c = 29$

**2004**

4 (b) (ii) Solve $z^2 - 10z + 26 = 0$.

Write your answers in the form $a + bi$, where $a, b \in \mathbf{R}$.

**SOLUTION**

$z^2 - 10z + 26 = 0$

$\therefore z = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(26)}}{2(1)}$

$= \dfrac{10 \pm \sqrt{100 - 104}}{2} = \dfrac{10 \pm \sqrt{-4}}{2} = \dfrac{10 \pm 2i}{2}$

$= 5 \pm i$

$z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ....... **3**

$a = 1$
$b = -10$
$c = 26$

**2002**

4 (c) $p$ and $k$ are real numbers such that $p(2+i)+8-ki=5k-3-i$.

(i) Find the value of $p$ and the value of $k$.

(ii) Investigate if $p + ki$ is a root of the equation $z^2-4z+13=0$.

**SOLUTION**

**4 (c) (i)**

For all equations you can equate (set equal) the real parts and the imaginary parts.

$p(2+i)+8-ki=5k-3-i$

$\Rightarrow 2p+pi+8-ki=5k-3-i$

$\Rightarrow (2p+8)+(p-k)i=(5k-3)-i$

Equate the real parts: $2p+8=5k-3 \Rightarrow 2p-5k=-11$....(**1**)

Equate the imaginary parts: $p-k=-1$....(**2**)

Solve equations (**1**) and (**2**) simultaneously.

$2p-5k=-11$....(**1**)

$p-k=-1$......(**2**)$(\times -2)$

→

$2p-5k=-11$

$-2p+2k=2$

$-3k=-9 \Rightarrow k=3$

Substitute this value into Eqn. (**2**): $p-(3)=-1 \Rightarrow p-3=-1 \Rightarrow p=2$

**4 (c) (ii)**

$p+ki=2+3i$

To show 2 + 3$i$ is a root of $z^2-4z+13=0$, substitute 2 + 3$i$ in for $z$. If you get an answer of zero, it is a root.

$z^2-4z+13$

$=(2+3i)^2-4(2+3i)+13$

$=4+12i+9i^2-8-12i+13$

$=4-9-8+13=0$

Therefore, 2 + 3$i$ is a root.

**1998**

4 (b) (i) Verify that $4-3i$ is a root of

$$z^2-8z+25=0$$

and write down the other root.

**SOLUTION**

If $a+bi$ is a root of a quadratic equation with all real coefficients, then its conjugate, $a-bi$, is also a root.

To show that that $4-3i$ is a root of $z^2-8z+25=0$ substitute it in for $z$ and show that you get zero.

$(4-3i)^2-8(4-3i)+25$

$=(4-3i)(4-3i)-8(4-3i)+25$

$=16-12i-12i+9i^2-32+24i+25$

$=16-12i-12i-9-32+24i+25$

$=0$

Therefore, $4-3i$ is a root. The other root is $4+3i$.

**1996**

4 (c) Let $z=2-i$ be one root of the equation $z^2+pz+q=0$, $p, q \in \mathbf{R}$.

Find the value of $p$ and the value of $q$.

**SOLUTION**

If $a+bi$ is a root of a quadratic equation with all real coefficients, then its conjugate, $a-bi$, is also a root.

If $2-i$ is a root of $z^2+pz+q=0$, then $2+i$ is also a root.

Substitute one of the roots into the quadratic.

$(2-i)^2+p(2-i)+q=0$

$\Rightarrow (2-i)(2-i)+p(2-i)+q=0$

$\Rightarrow 4-2i-2i+i^2+2p-pi+q=0$

$\Rightarrow 4-4i-1+2p-pi+q=0$

$\Rightarrow (3+2p+q)+(-p-4)i=0+0i$ [Gather up the real parts and the imaginary parts.]

Equate the imaginary parts: $-p-4=0 \Rightarrow p=-4$

Equate the real parts: $3+2p+q=0 \Rightarrow 3+2(-4)+q=0 \Rightarrow 3-8+q=0$

$\Rightarrow -5+q=0 \Rightarrow q=5$