Complex Numbers (Q 4, Paper 1)

Lesson No. 4: Division

2007

4 (c) Let $u = 3 + 2i$.

(i) Find the value of $u^2 + \bar{u}^2$, where $\bar{u}$ is the complex conjugate of u.

(ii) Investigate whether $\frac{13}{u} = \bar{u}$.

Solution

4 (c) (i)

Working out the conjugate: $z = a + bi \Rightarrow \bar{z} = a - bi$ **1**

$u = 3 + 2i \Rightarrow \bar{u} = 3 - 2i$

$\therefore u^2 + \bar{u}^2 = (3+2i)^2 + (3-2i)^2$

$= 9 + 12i + 4i^2 + 9 - 12i + 4i^2$ [Use the fact that $i^2 = -1$.]

$= 9 - 4 + 9 - 4 = 10$

4 (c) (ii)

Division: Multiply above and below by the conjugate of the bottom.

$$\frac{13}{u} = \frac{13}{3+2i}$$

$$= \frac{13}{(3+2i)} \times \frac{(3-2i)}{(3-2i)}$$ [Multiply above and below of the conjugate of the bottom.]

$$= \frac{39 - 26i}{9 - 6i + 6i - 4i^2} = \frac{39 - 26i}{9 + 4} = \frac{39 - 26i}{13}$$ [Divide the 13 into each term above.]

$$= 3 - 2i = \bar{u}$$

2004

4 (b) (i) Let $w = 1 - 2i$.

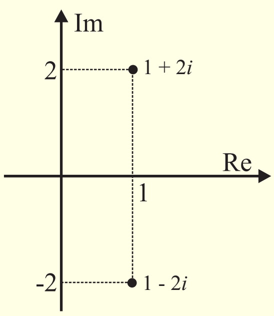
Plot w and $\bar{w}$ on an Argand diagram, where $\bar{w}$ is the complex conjugate of w.

Solution

Working out the conjugate:

$z = a + bi \Rightarrow \bar{z} = a - bi$ **1**

$w = 1 - 2i \Rightarrow \bar{w} = 1 + 2i$



2002

4 (b) Let $z = 5 + 4i$.

(i) Plot z and $\bar{z}$ on an Argand diagram, where $\bar{z}$ is the complex conjugate of z.

(ii) Calculate $z\bar{z}$.

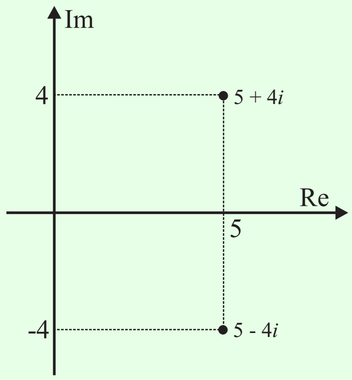
(iii) Express $\frac{z}{\bar{z}}$ in the form $u + vi$ where $u, v \in \mathbf{R}$.

SOLUTION

4 (b) (i) Working out the conjugate:

$z = a + bi \Rightarrow \bar{z} = a - bi$ **1**

$z = 5 + 4i \Rightarrow \bar{z} = 5 - 4i$



4 (b) (ii)

$z\bar{z} = (5+4i)(5-4i)$ [Multiply out the brackets.]

$= 25 - 20i + 20i - 16i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 25 + 16$

$= 41$

4 (b) (iii)

DIVISION: Multiply above and below by the conjugate of the bottom.

$\frac{z}{\bar{z}} = \frac{5+4i}{5-4i}$

$= \frac{(5+4i)}{(5-4i)} \times \frac{(5+4i)}{(5+4i)}$ [Multiply above and below by the conjugate of the bottom.]

$= \frac{25 + 20i + 20i + 16i^2}{41}$ [Tidy up using the fact that $i^2 = -1$.]

$= \frac{25 + 40i - 16}{41}$

$= \frac{9 + 40i}{41}$

$= \frac{9}{41} + \frac{40}{41}i$