Complex Numbers (Q 4, Paper 1)

Lesson No. 3: Adding and Multiplying Complex Nos.

2007

4 (a) Given that $i^2 = -1$, simplify

$3(2-4i)+i(5-6i)$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

Solution

$3(2-4i)+i(5-6i)$ [Multiply out the brackets.]

$= 6-12i+5i-6i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 6-12i+5i+6$ [Add the real numbers together and the imaginary numbers together.]

$= 12-7i$

2005

4 (a) Let $u = 4-2i$, where $i^2 = -1$.

Plot

(i) u

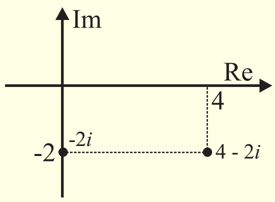
(ii) $u-4$

on an Argand diagram.

Solution

(i) Plot $u = 4-2i$

(ii) Plot $u-4 = 4-2i-4 = -2i$



2004

4 (a) Given that $i^2 = -1$, simplify

$4(2-i)+i(3+5i)$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

Solution

$4(2-i)+i(3+5i)$ [Multiply out the brackets.]

$= 8-4i+3i+5i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 8-4i+3i-5$

$= 3-i$

2002

4 (a) Given that $i^2 = -1$, simplify

$$2(3-i)+i(4+5i)$$

and write your answer in the form $x + yi$ where $x, y \in \mathbf{R}$.

SOLUTION

$2(3-i)+i(4+5i)$ [Multiply out the brackets.]

$= 6-2i+4i+5i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 6+2i-5$

$= 1+2i$

2001

4 (a) Let $w = 3-2i$ where $i^2 = -1$.
Plot
(i) w

(ii) iw
on an Argand diagram.

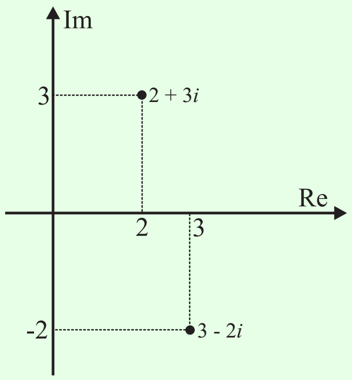
SOLUTION

4 (a) (i)

$w = 3-2i$

4 (a) (ii)

$iw = i(3-2i) = 3i-2i^2 = 2+3i$



2000

4 (a) Simplify

$$7(2+i)+i(11+9i)$$

and express your answer in the form $x + yi$ where $x, y \in \mathbf{R}$ and $i^2 = -1$.

SOLUTION

$7(2+i)+i(11+9i)$ [Multiply out the brackets.]

$= 14+7i+11i+9i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 14+18i-9$

$= 5+18i$

1999

4 (a) Let $z = 5 + 4i$, where $i^2 = -1$.
Plot
(i) z

(ii) $z - 4i$
on an Argand diagram.

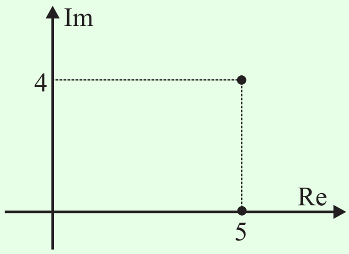
SOLUTION

4 (a) (i)

$z = 5 + 4i$

4 (a) (ii)

$z - 4i = 5 + 4i - 4i = 5 + 0i$



1997

4 (a) Simplify

$$3(1+5i)+i(3-2i)$$

and express your answer in the form $p + qi$, where $p, q \in \mathbf{R}$ and $i^2 = -1$.

(b) (ii) If $w = 4i$, verify that

$$w^3 - w^2 + 16w - 16 = 0.$$

SOLUTION

4 (a)

$3(1+5i)+i(3-2i)$ [Multiply out the brackets.]

$= 3+15i+3i-2i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 3+18i+2$ [Add the real numbers together and the imaginary numbers together.]

$= 5+18i$

4 (b) (ii)

$w^3 - w^2 + 16w - 16$

$= (4i)^3 - (4i)^2 + 16(4i) - 16$

$= 64i^3 - 16i^2 + 64i - 16$

$= -64i + 16 + 64i - 16$

$= 0$

Powers of i

$i = \sqrt{-1} = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

1996

4 (a) Let $z = 1 - 4i$, where $i^2 = -1$.
Plot z and $2 + z$ on an Argand diagram.

SOLUTION

$z = 1 - 4i$

$2 + z = 2 + 1 - 4i = 3 - 4i$