COMPLEX NUMBERS (Q 4, PAPER 1)

2011

4. (a) Let $u = 1 + 2i$, where $i^2 = -1$.
Plot on an Argand diagram
(i) u

(ii) $u - 3$.

(b) Let $z = 2 + 3i$.

(i) Find z^2 in the form $x + yi$, where $x, y \in \mathbb{R}$.

(ii) Show that $z^2 = 4z - 13$.

(iii) Show that $\bar{z}^2 + 13 = 4\bar{z}$, where $\bar{z}$ is the complex conjugate of z.

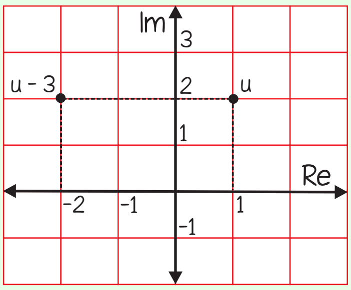
(c) (i) Express $\frac{4+2i}{3-i}$ in the form $x + yi$, where $x, y \in \mathbb{R}$.

(ii) Hence, or otherwise, find the real numbers k and t such that

$$\left|\frac{4+2i}{3-i}\right|(k+5i) = \frac{1}{\sqrt{2}}(7+(t-1)i).$$

SOLUTION

4 (a)



4 (b) (i)

$z = 2 + 3i$

$z^2 = (2+3i)(2+3i)$

$= 2(2+3i) + 3i(2+3i)$

$= 4 + 6i + 6i + 9i^2$

$= 4 + 6i + 6i + 9(-1)$

$= 4 + 6i + 6i - 9$

$= -5 + 12i$

4 (b) (ii)

$z = 2 + 3i$

$z^2 = -5 + 12i$

$$\begin{aligned} 4z - 13 &= 4(2+3i) - 13 \\ &= 8 + 12i - 13 \\ &= -5 + 12i \end{aligned}$$

$\therefore z^2 = 4z - 13$

4 (b) (iii)

$z = 2 + 3i \Rightarrow \bar{z} = 2 - 3i$

$z = a + bi \Rightarrow \bar{z} = a - bi$

$$\begin{aligned} \bar{z}^2 + 13 &= (2-3i)(2-3i) + 13 \\ &= [2(2-3i) - 3i(2-3i)] + 13 \\ &= 4 - 6i - 6i + 9i^2 + 13 \\ &= 4 - 6i - 6i + 9(-1) + 13 \\ &= 4 - 6i - 6i - 9 + 13 \\ &= 8 - 12i \end{aligned}$$

$$\begin{aligned} 4\bar{z} &= 4(2-3i) \\ &= 8 - 12i \end{aligned}$$

$\therefore \bar{z}^2 + 13 = 4\bar{z}$

4 (c) (i)

Division: Multiply above and below by the conjugate of the bottom.

$$\frac{4+2i}{3-i}$$

$$= \frac{(4+2i)}{(3-i)} \times \frac{(3+i)}{(3+i)}$$

$$= \frac{4(3+i) + 2i(3+i)}{3(3+i) - i(3+i)}$$

$$= \frac{12 + 4i + 6i + 2i^2}{9 + 3i - 3i - i^2}$$

$$= \frac{12 + 4i + 6i - 2}{9 + 3i - 3i + 1}$$

$$= \frac{10 + 10i}{10}$$

$$= 1 + i$$

4 (c) (ii)

$$\left|\frac{4+2i}{3-i}\right|(k+5i)=\frac{1}{\sqrt{2}}(7+(t-1)i)$$

$$|1+i|(k+5i)=\frac{1}{\sqrt{2}}(7+(t-1)i)$$

$z=a+bi \Rightarrow |z|=\sqrt{a^2+b^2}$

$$\sqrt{1^2+1^2}(k+5i)=\frac{1}{\sqrt{2}}(7+(t-1)i)$$

$$\sqrt{2}(k+5i)=\frac{1}{\sqrt{2}}(7+(t-1)i)$$

$$2(k+5i)=(7+(t-1)i)$$

$$2k+10i=(7+(t-1)i)$$

$$2k=7 \Rightarrow k=\tfrac{7}{2}$$

$$10=t-1 \Rightarrow t=11$$