COMPLEX NUMBERS (Q 4, PAPER 1)

2010

4. **(a)** Given that $i^2 = -1$, simplify

$$(4 + 2i)(3 - i)$$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

(b) Let $u = 4 + 3i$ and $w = 6 - 8i$.

(i) Find the value of the real number k such that $|u| = k|w|$.

(ii) Express $\frac{w}{u}$ in the form $x + yi$.

(c) Let $z = a + bi$, where $a, b \in \mathbf{R}$.
Find the value of a and the value of b for which

$$3z - 10i = (2 - 3i)z.$$

SOLUTION

4 (a)

$$(4+2i)(3-i)$$
$$= 12 - 4i + 6i - 2i^2$$
$$= 12 + 2i + 2$$
$$= 14 + 2i$$

4 (b) (i)

$$|u| = k|w|$$
$$|4+3i| = k|6-8i|$$
$$\sqrt{4^2+3^2} = k\sqrt{6^2+(-8)^2}$$

$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$

$$\sqrt{16+9} = k\sqrt{36+64}$$
$$\sqrt{25} = k\sqrt{100}$$
$$5 = 10k$$
$$\therefore k = \tfrac{1}{2}$$

4 (c) (ii)

$$|u| = k|w|$$
$$|4+3i| = k|6-8i|$$
$$\frac{w}{u} = \frac{6-8i}{4+3i}$$
$$= \frac{(6-8i)}{(4+3i)} \times \frac{(4-3i)}{(4-3i)}$$
$$= \frac{24-18i-32i+24i^2}{16-12i+12i-9i^2}$$
$$= \frac{24-50i-24}{16+9}$$
$$= \frac{0-50i}{25} = 0-2i$$

Working out the conjugate:

$z = a+bi \Rightarrow \bar{z} = a-bi$

DIVISION: Multiply above and below by the conjugate of the bottom.

4 (c) (ii)

$$3z-10i = (2-3i)z$$
$$3(a+bi)-10i = (2-3i)(a+bi)$$
$$3a+3bi-10i = 2a+2bi-3ai-3bi^2$$
$$3a+3bi-10i = 2a+2bi-3ai+3b$$
$$3a+(3b-10)i = (2a+3b)+(2b-3a)i$$
$$\therefore 3a = 2a+3b \Rightarrow a = 3b.....(\mathbf{1})$$
$$\therefore 3b-10 = 2b-3a \Rightarrow b = 10-3a.....(\mathbf{2})$$

For all equations you can equate (set equal) the real parts and the imaginary parts.

Solve equations (**1**) and (**2**) for a and b. Replace a in Eqn. (**2**) by its value in Eqn. (**1**).

$$b = 10-3a \Rightarrow b = 10-3(3b)$$
$$b = 10-9b$$
$$10b = 10$$
$$\therefore b = 1$$

Substitute this value of b into Eqn. (**1**) to find a.

$$a = 3b \Rightarrow a = 3(1)$$
$$\therefore a = 3$$