

COMPLEX NUMBERS (Q 4, PAPER 1)

2009

4 (a) Given that $i^2 = -1$, simplify

$$2(3 - 5i) + 7i(2 + 3i)$$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

(b) Let $u = 3 + 5i$.

(i) Show that u is a solution of the equation $z^2 - 6z + 34 = 0$.

(ii) Express $\frac{17}{u}$ in the form $x + yi$.

(c) Let $z = 3 - 4i$.

(i) Calculate $|z|$.

(ii) Find the real numbers p and q such that

$$|z|(p + qi) + (q - pi) = 17 + 7i.$$

SOLUTION

4 (a)

$$\begin{aligned} & 2(3 - 5i) + 7i(2 + 3i) \\ &= 6 - 10i + 14i + 21i^2 \\ &= 6 + 4i - 21 \\ &= -15 + 4i \end{aligned}$$

4 (b) (i)

Substitute the root, $(3 + 5i)$, into the equation. If you get zero you have proved that $(3 + 5i)$, is a solution of this equation.

$$\begin{aligned} & z^2 - 6z + 34 \\ &= (3 + 5i)^2 - 6(3 + 5i) + 34 \\ &= (3 + 5i)(3 + 5i) - 6(3 + 5i) + 34 \\ &= 9 + 15i + 15i + 25i^2 - 18 - 30i + 34 \\ &= 9 + 15i + 15i - 25 - 18 - 30i + 34 \\ &= 0 \end{aligned}$$

4 (b) (ii)

$$\begin{aligned} \frac{17}{u} &= \frac{17}{3+5i} \\ &= \frac{17}{(3+5i)} \times \frac{(3-5i)}{(3-5i)} \\ &= \frac{17(3-5i)}{9-15i+15i-25i^2} = \frac{17(3-5i)}{9+25} \\ &= \frac{17(3-5i)}{34} = \frac{(3-5i)}{2} \\ &= \frac{3}{2} - \frac{5}{2}i \end{aligned}$$

Working out the conjugate:

$$z = a + bi \Rightarrow \bar{z} = a - bi$$

DIVISION: Multiply above and below by the conjugate of the bottom.

4 (c) (i)

$$\begin{aligned} z = 3 - 4i \Rightarrow |z| &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$$

4 (c) (ii)

$$\begin{aligned} |z|(p + qi) + (q - pi) &= 17 + 7i \\ 5(p + qi) + (q - pi) &= 17 + 7i \\ 5p + 5qi + q - pi &= 17 + 7i \\ (5p + q) + (5q - p)i &= 17 + 7i \\ \therefore 5p + q &= 17 \dots (1) \\ \therefore -p + 5q &= 7 \dots (2) \end{aligned}$$

Solve Equations (1) and (2) simultaneously.

$$\begin{aligned} 5p + q &= 17 \dots (1) \\ -p + 5q &= 7 \dots (2) \times 5 \end{aligned}$$

$$\begin{array}{r} 5p + q = 17 \\ -5p + 25q = 35 \\ \hline 26q = 52 \Rightarrow q = 2 \end{array}$$

Substitute this value of q into Eqn. (1) to find p .

$$\begin{aligned} 5p + q = 17 &\Rightarrow 5p + (2) = 17 \\ 5p &= 15 \\ \therefore p &= 3 \end{aligned}$$