COMPLEX NUMBERS (Q 4, PAPER 1)

2008

4 (a) Let $u = 3 - 4i$, where $i^2 = -1$.
Plot on an argand diagram
(i) u

(ii) $u + 5i$.

(b) Let $w = 2 + 5i$.
(i) Express w^2 in the form $x + yi$, $x, y \in \mathbf{R}$.

(ii) Verify that $|w^2| = |w|^2$.

(c) Let $z = 6 - 4i$.
(i) Find the real number k such that

$$k(z + \bar{z}) = 24$$

where $\bar{z}$ is the complex conjugate of z.

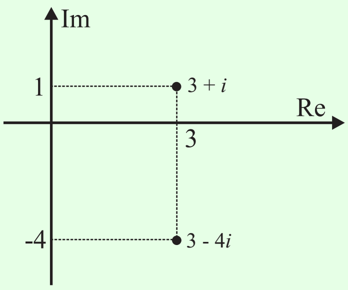
(ii) Find the real numbers s and t such that

$$\frac{s + ti}{4 + 3i} = z.$$

SOLUTION

4 (a) (i)

$u = 3 - 4i$



4 (a) (ii)

$u + 5i = 3 - 4i + 5i = 3 + i$

4 (b) (i)

$w^2 = (2 + 5i)^2 = (2 + 5i)(2 + 5i)$

$\Rightarrow w^2 = 4 + 10i + 10i + 25i^2$

$\Rightarrow w^2 = 4 + 20i - 25$

$\therefore w^2 = -21 + 20i$

4 (b) (ii)

$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$ **2**

$|w^2| = |-21 + 20i| = \sqrt{(-21)^2 + (20)^2} = \sqrt{441 + 400} = \sqrt{841} = 29$

$|w|^2 = |2 + 5i|^2 = (\sqrt{2^2 + 5^2})^2 = 4 + 25 = 29$

4 (c) (i)

$k(z+\bar{z})=24$

$\Rightarrow k(6-4i+6+4i)=24$

$\Rightarrow 12k=24$

$\therefore k=2$

Working out the conjugate:

$z=a+bi \Rightarrow \bar{z}=a-bi$ 1

4 (c) (ii)

$\frac{s+ti}{4+3i}=z \Rightarrow \frac{s+ti}{4+3i}=6-4i$

$\Rightarrow s+ti=(6-4i)(4+3i)$

$\Rightarrow s+ti=24+18i-16i-12i^2$

$\Rightarrow s+ti=24+2i+12$

$\therefore s+ti=36+2i$

$\therefore s=36,\ t=2$