COMPLEX NUMBERS (Q 4, PAPER 1)

2007

4 (a) Given that $i^2 = -1$, simplify

$3(2-4i)+i(5-6i)$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

(b) Let $z = 5-3i$.

(i) Plot z and $-z$ on an Argand diagram.

(ii) Calculate $|z-1|$.

(iii) Find the value of the real number k such that $ki + 4z = 20$.

(c) Let $u = 3 + 2i$.

(i) Find the value of $u^2 + \bar{u}^2$, where $\bar{u}$ is the complex conjugate of u.

(ii) Investigate whether $\frac{13}{u} = \bar{u}$.

SOLUTION

4 (a)

$3(2-4i)+i(5-6i)$ [Multiply out the brackets.]

$= 6-12i+5i-6i^2$ [Use the fact that $i^2 = -1$.]

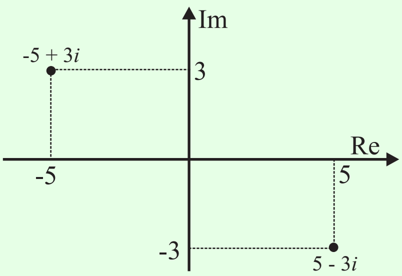
$= 6-12i+5i+6$ [Add the real numbers together and the imaginary numbers together.]

$= 12-7i$

4 (b) (i)

$z = 5-3i$

$-z = -5+3i$



4 (b) (ii)

Finding the modulus:

$z = a+bi \Rightarrow |z| = \sqrt{a^2+b^2}$ 2

$|z-1| = |5-3i-1| = |4-3i|$

$= \sqrt{(4)^2+(-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

4 (b) (iii)

For all equations you can equate (set equal) the real parts and the imaginary parts.

$ki + 4z = 20$

$\Rightarrow ki + 4(5 - 3i) = 20$

$\Rightarrow ki + 20 - 12i = 20$

$\Rightarrow ki - 12i = 0$

$\Rightarrow 0 + (k - 12)i = 0 + 0i$

$\therefore k - 12 = 0 \Rightarrow k = 12$

4 (c) (i)

Working out the conjugate: $z = a + bi \Rightarrow \bar{z} = a - bi$ 1

$u = 3 + 2i \Rightarrow \bar{u} = 3 - 2i$

$\therefore u^2 + \bar{u}^2 = (3 + 2i)^2 + (3 - 2i)^2$

$= 9 + 12i + 4i^2 + 9 - 12i + 4i^2$ [Use the fact that $i^2 = -1$.]

$= 9 - 4 + 9 - 4 = 10$

4 (c) (ii)

Division: Multiply above and below by the conjugate of the bottom.

$$\frac{13}{u} = \frac{13}{3 + 2i}$$

$$= \frac{13}{(3 + 2i)} \times \frac{(3 - 2i)}{(3 - 2i)}$$ [Multiply above and below of the conjugate of the bottom.]

$$= \frac{39 - 26i}{9 - 6i + 6i - 4i^2} = \frac{39 - 26i}{9 + 4} = \frac{39 - 26i}{13}$$ [Divide the 13 into each term above.]

$= 3 - 2i = \bar{u}$