COMPLEX NUMBERS (Q 4, PAPER 1)

2006

4 (a) Let $u = 3 - 6i$ where $i^2 = -1$.
Calculate $|u + 2i|$.

(b) (i) Solve $z^2 - 4z + 29 = 0$.
Write your answers in the form $x + yi$ where $x, y \in \mathbf{R}$.

(ii) Write in its simplest form $i(i^4 + i^5 + i^6)$.

(c) (i) Express $\dfrac{3-2i}{1-4i}$ in the form $x + yi$.

(ii) Hence, or otherwise, find the values of the real numbers p and q such that

$$p + 2qi = \frac{17(3-2i)}{1-4i}.$$

SOLUTION

4 (a)

Finding the modulus:

$$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$$ 2

$$|u + 2i| = |3 - 6i + 2i| = |3 - 4i|$$

$$= \sqrt{(3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

4 (b) (i)

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ 3

$a = 1$
$b = -4$
$c = 29$

$$z^2 - 4z + 29 = 0$$

$$\therefore z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(29)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 116}}{2} = \frac{4 \pm \sqrt{-100}}{2}$$

$$= \frac{4 \pm 10i}{2} = 2 \pm 5i$$

4 (b) (ii)

$$i(i^4 + i^5 + i^6)$$

$$= i^5 + i^6 + i^7$$

$$= i + i^2 + i^3$$

$$= i - 1 - i = -1$$

Powers of i
$i = \sqrt{-1} = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

$i^{\text{power}} = i^{\text{remainder when } \textit{power} \text{ is divided by 4}}$

When you see a power of i, divide the power by 4 and take the remainder. Now use the table on the left to write your answer.

Powers of i repeat in groups of four. You always get one of 4 answers: $i, -1, -i, 1$

4 (c) (i)

Division: Multiply above and below by the conjugate of the bottom.

$\frac{3-2i}{1-4i}$ [Multiply above and below by the conjugate of the bottom.]

$=\frac{(3-2i)}{(1-4i)}\times\frac{(1+4i)}{(1+4i)}$ [Multiply out the brackets.]

$=\frac{3+12i-2i-8i^2}{1+4i-4i-16i^2}$ [Tidy up using the fact that $i^2=-1$.]

$=\frac{3+10i+8}{1+16}=\frac{11+10i}{17}$ [Divide the 17 into each term on top.]

$=\frac{11}{17}+\frac{10}{17}i$

4 (c) (ii)

For all equations you can equate (set equal) the real parts and the imaginary parts.

$p+2qi=\frac{17(3-2i)}{1-4i}$

$\Rightarrow p+2qi=\frac{17(11+10i)}{17}$

$\Rightarrow p+2qi=11+10i$ [Equate the real parts and the imaginary parts.]

$\therefore p=11$ and $2q=10\Rightarrow q=5$