COMPLEX NUMBERS (Q 4, PAPER 1)

2005

4 (a) Let $u = 4 - 2i$, where $i^2 = -1$.
Plot
(i) u
(ii) $u - 4$
on an Argand diagram.

(b) Let $w = 1 + 3i$.
(i) Express $\frac{2}{w}$ in the form $x + yi$, where $x, y \in \mathbf{R}$.

(ii) Investigate whether $|iw + w| = |iw| + |w|$.

(c) Let $z = 1 - 2i$.
(i) Write down $\overline{z}$, the complex conjugate of z.

(ii) Find the real numbers k and t such that

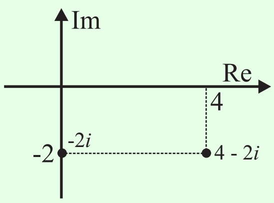
$$kz + t\overline{z} = 2z^2.$$

SOLUTION

4 (a)

(i) Plot $u = 4 - 2i$

(ii) Plot $u - 4 = 4 - 2i - 4 = -2i$



4 (b) (i)

$w = 1 = 3i$

DIVISION: Multiply above and below by the conjugate of the bottom.

$\frac{2}{w} = \frac{2}{1+3i}$ [Multiply above and below by the conjugate.]

$= \frac{2}{(1+3i)} \times \frac{(1-3i)}{(1-3i)}$ [Multiply out the brackets.]

$= \frac{2-6i}{1-3i+3i-9i^2}$ [Tidy up the bottom using the fact that $i^2 = -1$.]

$= \frac{2-6i}{1+9}$ [Divide the bottom number into each term on top.]

$= \frac{2-6i}{10} = \frac{1}{5} - \frac{3}{5}i$

4 (b) (ii)

Finding the modulus:

$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$ **2**

LHS	*RHS*
$\lvert iw + w\rvert = \lvert i(1+3i) + (1+3i)\rvert$	$\lvert iw\rvert + \lvert w\rvert = \lvert i(1+3i)\rvert + \lvert 1+3i\rvert$
$= \lvert i + 3i^2 + 1 + 3i\rvert$	$= \lvert i + 3i^2\rvert + \lvert 1+3i\rvert$
$= \lvert i - 3 + 1 + 3i\rvert$	$= \lvert -3+i\rvert + \lvert 1+3i\rvert$
$= \lvert -2 + 4i\rvert$	$= \sqrt{(-3)^2 + (1)^2} + \sqrt{(1)^2 + (3)^3}$
$= \sqrt{(-2)^2 + (4)^2}$	$= \sqrt{10} + \sqrt{10}$
$= \sqrt{4+16} = \sqrt{20}$	$= 2\sqrt{10}$
$= 2\sqrt{5}$	

Therefore, this statement is false.

4 (c) (i)

Working out the conjugate: $z = a + bi \Rightarrow \bar{z} = a - bi$ **1**

$z = 1 - 2i \Rightarrow \bar{z} = 1 + 2i$

4 (c) (ii)

For all equations you can equate (set equal) the real parts and the imaginary parts.

$kz + t\bar{z} = 2z^2 \Rightarrow k(1-2i) + t(1+2i) = 2(1-2i)^2$ [Multiply out the brackets.]

$\Rightarrow k - 2ki + t + 2ti = 2(1 - 4i + 4i^2)$ [Gather up the real and imaginary parts on the left.]

$\Rightarrow (k+t) + (2t - 2k)i = 2(1 - 4i - 4)$

$\Rightarrow (k+t) + (2t - 2k)i = 2(-3 - 4i)$

$\Rightarrow (k+t) + (2t - 2k)i = -6 - 8i$ [Equate the real parts and the imaginary parts.]

Equating the real parts: $k + t = -6$....**(1)**

Equating the imaginary parts: $2t - 2k = -8 \Rightarrow t - k = -4$....**(2)**

Solve Equations (**1**) and (**2**) simultaneously.

$t + k = -6$....**(1)**

$t - k = -4$....**(2)**

$2t \quad = -10 \Rightarrow t = -5$

Substitute this value of t into Eqn. (**1**): $(-5) + k = -6 \Rightarrow k = -6 + 5 \Rightarrow k = -1$