Complex Numbers (Q 4, Paper 1)

2004

4 (a) Given that $i^2 = -1$, simplify

$4(2-i)+i(3+5i)$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

(b) (i) Let $w = 1-2i$.

Plot w and $\bar{w}$ on an Argand diagram, where $\bar{w}$ is the complex conjugate of w.

(ii) Solve $z^2 - 10z + 26 = 0$.

Write your answers in the form $a + bi$, where $a, b \in \mathbf{R}$.

(c) Let $z_1 = 5+12i$ and $z_2 = 2-3i$.

(i) Find the value of the real number k such that $|z_1| = k|z_2|$.

(ii) p and q are real numbers such that

$$\frac{z_1}{z_2} = p(q+i).$$

Find the value of p and the value of q.

Solution

4 (a)

$4(2-i)+i(3+5i)$ [Multiply out the brackets.]

$= 8-4i+3i+5i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 8-4i+3i-5$

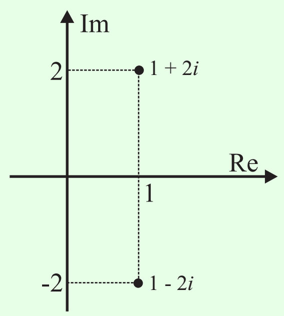
$= 3-i$

4 (b) (i)

Working out the conjugate:

$z = a+bi \Rightarrow \bar{z} = a-bi$ **1**

$w = 1-2i \Rightarrow \bar{w} = 1+2i$



4 (b) (ii)

$z^2 - 10z + 26 = 0$

$z = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ **3**

$a = 1$
$b = -10$
$c = 26$

$\therefore z = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(26)}}{2(1)}$

$= \dfrac{10 \pm \sqrt{100-104}}{2} = \dfrac{10 \pm \sqrt{-4}}{2} = \dfrac{10 \pm 2i}{2}$

$= 5 \pm i$

4 (c) (i)

Finding the modulus:

$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$ **2**

$|z_1| = k|z_2| \Rightarrow |5+12i| = k|2-3i|$

$\Rightarrow \sqrt{(5)^2 + (12)^2} = k\sqrt{(2)^2 + (-3)^2}$

$\Rightarrow \sqrt{25+144} = k\sqrt{4+9}$

$\Rightarrow \sqrt{169} = k\sqrt{13}$

$\Rightarrow 13 = k\sqrt{13} \Rightarrow k = \dfrac{13}{\sqrt{13}} = \sqrt{13}$

4 (c) (ii)

Division: Multiply above and below by the conjugate of the bottom.

$\dfrac{z_1}{z_2} = p(q+i) \Rightarrow \dfrac{5+12i}{2-3i} = pq + pi$

$\Rightarrow \dfrac{(5+12i)}{(2-3i)} \times \dfrac{(2+3i)}{(2+3i)} = pq + pi$ [Multiply the left hand side above and below by the conjugate of the bottom.]

$\Rightarrow \dfrac{10+15i+24i+36i^2}{4+6i-6i-9i^2} = pq + pi$ [Tidy up the left hand side using the fact that $i^2 = -1$.]

$\Rightarrow \dfrac{10+39i-36}{4+9} = pq + pi$

$\Rightarrow \dfrac{-26+39i}{13} = pq + pi$

$\Rightarrow -2+3i = pq + pi$ [Equate the real parts and the imaginary parts.]

For all equations you can equate (set equal) the real parts and the imaginary parts.

Equate the imaginary parts: $\therefore\ p = 3$

Equate the real parts: $\therefore\ pq = -2 \Rightarrow (3)q = -2 \Rightarrow q = -\frac{2}{3}$