COMPLEX NUMBERS (Q 4, PAPER 1)

2003

4 (a) Given that $i^2 = -1$, find the value of:

(i) i^8

(ii) i^7.

(b) Let $z_1 = 2+3i$ and $z_2 = 5-i$.

(i) Plot z_1 and z_2 and $z_1 + z_2$ on an Argand diagram.

(ii) Investigate whether $|z_1 + z_2| > |z_1 - z_2|$.

(c) Let $w = 1 + i$.

(i) Simplify $\frac{6}{w}$.

(ii) a and b are real numbers such that

$$a\left(\frac{6}{w}\right) - b(w+1) = 3(w+i).$$

Find the value of a and the value of b.

SOLUTION

Powers of i

$i = \sqrt{-1} = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^{power} = i^{\text{remainder when } power \text{ is divided by 4}}$

When you see a power of i, divide the power by 4 and take the remainder. Now use the table on the left to write your answer.

Powers of i repeat in groups of four. You always get one of 4 answers: $i, -1, -i, 1$

4 (a) (i)

$i^8 = i^0 = 1$

4 (a) (ii)

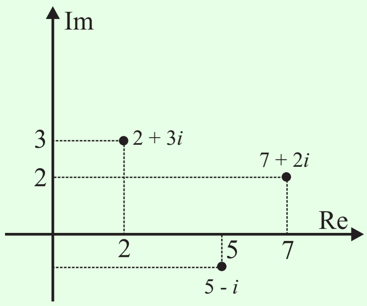
$i^7 = i^3 = -i$

4 (b) (i)

$z_1 = 2+3i$

$z_2 = 5-i$

$z_1 + z_2 = 2+3i+5-i = 7+2i$



4 (b) (ii)

Finding the modulus:

$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$ **2**

LHS

$|z_1 + z_2| = |7 + 2i|$

$= \sqrt{7^2 + 2^2} = \sqrt{49 + 4}$

$= \sqrt{53}$

RHS

$|z_1 - z_2| = |2 + 3i - 5 + i|$

$= |-3 + 4i|$

$= \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16}$

$= \sqrt{25}$

As $\sqrt{53} > \sqrt{25}$, the statement is true.

4 (c) (i)

Working out the conjugate: $z = a + bi \Rightarrow \bar{z} = a - bi$ **1**

Division: Multiply above and below by the conjugate of the bottom.

$\frac{6}{w} = \frac{6}{1+i}$

$= \frac{6}{(1+i)} \times \frac{(1-i)}{(1-i)}$ [Multiply above and below by the conjugate of the bottom.]

$= \frac{6-6i}{1-i+i-i^2}$ [Tidy up using the fact that $i^2 = -1$.]

$= \frac{6-6i}{1+1} = \frac{6-6i}{2}$

$= 3 - 3i$

4 (c) (ii)

$a\left(\frac{6}{w}\right) - b(w+1) = 3(w+i)$

$\Rightarrow a(3-3i) - b(1+i+1) = 3(1+i+i)$

$\Rightarrow a(3-3i) - b(2+i) = 3(1+2i)$

$\Rightarrow 3a - 3ai - 2b - bi = 3 + 6i$

$\Rightarrow 3a - 2b + (-3a - b)i = 3 + 6i$

For all equations you can equate (set equal) the real parts and the imaginary parts.

Equating the real parts: $3a - 2b = 3 ...(\mathbf{1})$

Equating the imaginary parts: $-3a - b = 6 ...(\mathbf{2})$

Solve the equations (**1**) and (**2**) simultaneously.

$3a - 2b = 3 ...(\mathbf{1})$

$-3a - b \;\; = 6 ...(\mathbf{2})(\times -2)$

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$3a - 2b = 3$

$6a + 2b = -12$

$9a \quad = -9 \Rightarrow a = -1$

Substitute this value of *a* into Eqn. (**1**): $3(-1) - 2b = 3 \Rightarrow -3 - 2b = 3 \Rightarrow -2b = 6 \Rightarrow b = -3$