COMPLEX NUMBERS (Q 4, PAPER 1)

2002

4 (a) Given that $i^2 = -1$, simplify

$$2(3-i)+i(4+5i)$$

and write your answer in the form $x + yi$ where $x, y \in \mathbf{R}$.

(b) Let $z = 5 + 4i$.

(i) Plot z and $\bar{z}$ on an Argand diagram, where $\bar{z}$ is the complex conjugate of z.

(ii) Calculate $z\bar{z}$.

(iii) Express $\frac{z}{\bar{z}}$ in the form $u + vi$ where $u, v \in \mathbf{R}$.

(c) p and k are real numbers such that $p(2+i)+8-ki = 5k-3-i$.

(i) Find the value of p and the value of k.

(ii) Investigate if $p + ki$ is a root of the equation $z^2 - 4z + 13 = 0$.

SOLUTION

4 (a)

$2(3-i)+i(4+5i)$ [Multiply out the brackets.]

$= 6-2i+4i+5i^2$ [Tidy up using the fact that $i^2 = -1$.]

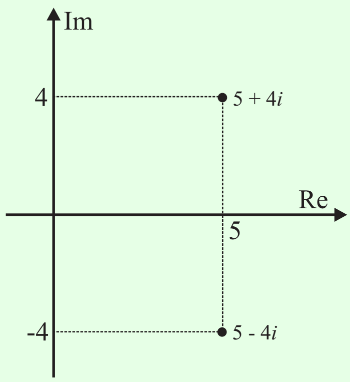
$= 6+2i-5$

$= 1+2i$

4 (b) (i) Working out the conjugate:

$z = a+bi \Rightarrow \bar{z} = a-bi$ 1

$z = 5+4i \Rightarrow \bar{z} = 5-4i$



4 (b) (ii)

$z\bar{z} = (5+4i)(5-4i)$ [Multiply out the brackets.]

$= 25-20i+20i-16i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 25+16$

$= 41$

4 (b) (iii)

Division: Multiply above and below by the conjugate of the bottom.

$$\frac{z}{\bar{z}} = \frac{5+4i}{5-4i}$$

$$= \frac{(5+4i)}{(5-4i)} \times \frac{(5+4i)}{(5+4i)}$$ [Multiply above and below by the conjugate of the bottom.]

$$= \frac{25+20i+20i+16i^2}{41}$$ [Tidy up using the fact that $i^2 = -1$.]

$$= \frac{25+40i-16}{41}$$

$$= \frac{9+40i}{41}$$

$$= \frac{9}{41} + \frac{40}{41}i$$

4 (c) (i)

For all equations you can equate (set equal) the real parts and the imaginary parts.

$$p(2+i)+8-ki = 5k-3-i$$

$$\Rightarrow 2p+pi+8-ki = 5k-3-i$$

$$\Rightarrow (2p+8)+(p-k)i = (5k-3)-i$$

Equate the real parts: $2p+8 = 5k-3 \Rightarrow 2p-5k = -11....(\mathbf{1})$

Equate the imaginary parts: $p-k = -1....(\mathbf{2})$

Solve equations (**1**) and (**2**) simultaneously.

$$2p-5k = -11....(\mathbf{1})$$
$$p-k = -1......(\mathbf{2})(\times -2)$$

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$$2p-5k = -11$$
$$-2p+2k = 2$$
$$-3k = -9 \Rightarrow k = 3$$

Substitute this value into Eqn. (**2**): $p-(3) = -1 \Rightarrow p-3 = -1 \Rightarrow p = 2$

4 (c) (ii)

$$p+ki = 2+3i$$

To show 2 + 3*i* is a root of $z^2-4z+13=0$, substitute 2 + 3*i* in for *z*. If you get an answer of zero, it is a root.

$$z^2-4z+13$$

$$= (2+3i)^2 - 4(2+3i)+13$$

$$= 4+12i+9i^2-8-12i+13$$

$$= 4-9-8+13 = 0$$

Therefore, 2 + 3*i* is a root.