COMPLEX NUMBERS (Q 4, PAPER 1)

2001

4 (a) Let $w = 3 - 2i$ where $i^2 = -1$.
Plot
(i) w

(ii) iw
on an Argand diagram.

(b) Solve
$$(x + 2yi)(1 - i) = 7 + 5i$$
for real x and for real y.

(c) Let $z_1 = 3 + 4i$ and $z_2 = 12 - 5i$.
$\bar{z}_1$ and $\bar{z}_2$ are the complex conjugates of z_1 and z_2, respectively.
(i) Show that $z_1\bar{z}_2 + \bar{z}_1 z_2$ is a real number.

(ii) Investigate if $|z_1| + |z_2| = |z_1 + z_2|$.

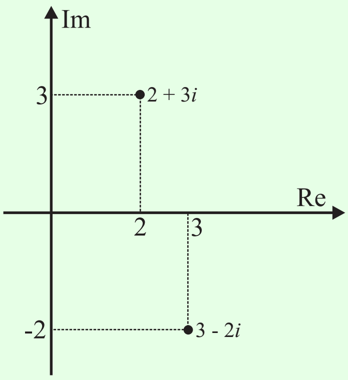
SOLUTION

4 (a) (i)

$w = 3 - 2i$

4 (a) (ii)

$iw = i(3 - 2i) = 3i - 2i^2 = 2 + 3i$



4 (b)

For all equations you can equate (set equal) the real parts and the imaginary parts.

$(x + 2yi)(1 - i) = 7 + 5i$ [Multiply out the brackets.]

$\Rightarrow x - xi + 2yi - 2yi^2 = 7 + 5i$ [Tidy up using the fact that $i^2 = -1$.]

$\Rightarrow x - xi + 2yi + 2y = 7 + 5i$ [Gather up the real parts and the imaginary parts.]

$\Rightarrow (x + 2y) + (-x + 2y)i = 7 + 5i$ [Equate the real parts and the imaginary parts.]

Equating the real parts: $x + 2y = 7$...(**1**)

Equating the imaginary parts: $-x + 2y = 5$....(**2**)

Solve equations (**1**) and (**2**) simultaneously.

$$x + 2y = 7 \text{...}(\mathbf{1})$$
$$-x + 2y = 5 \text{....}(\mathbf{2})$$
$$4y = 12 \Rightarrow y = 3$$

Substitute this value for y into Eqn. (**1**):

$x + 2(3) = 7 \Rightarrow x + 6 = 7 \Rightarrow x = 1$

4 (c) (i)

Working out the conjugate: $z = a + bi \Rightarrow \bar{z} = a - bi$ 1

$z_1\bar{z}_2 + \bar{z}_1 z_2$

$= (3+4i)(12+5i) + (3-4i)(12-5i)$ [Multiply out the brackets.]

$= 36 + 15i + 48i + 20i^2 + 36 - 15i - 48i + 20i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 36 - 20 + 36 - 20$

$= 32$

4 (c) (ii)

Finding the modulus:

$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$ 2

LHS

$|z_1| + |z_2|$

$= |3+4i| + |12-5i|$

$= \sqrt{3^2 + 4^2} + \sqrt{12^2 + (-5)^2}$

$= \sqrt{9+16} + \sqrt{144+25}$

$= \sqrt{25} + \sqrt{169}$

$= 5 + 13$

$= 18$

RHS

$|z_1 + z_2|$

$= |3 + 4i + 12 - 5i|$

$= |15 - i|$

$= \sqrt{15^2 + (-1)^2}$

$= \sqrt{225 + 1}$

$= \sqrt{226}$

$= 15.03$

Therefore, the statement is not true.