COMPLEX NUMBERS (Q 4, PAPER 1)

2000

4 (a) Simplify

$7(2+i)+i(11+9i)$

and express your answer in the form $x + yi$ where $x, y \in \mathbf{R}$ and $i^2 = -1$.

(b) Let $w = 3 - i$.

(i) Plot w and $w + 6i$ on an Argand diagram.

(ii) Calculate $|w + 6i|$.

(iii) Express $\frac{1}{w+6i}$ in the form $u + vi$ where $u, v \in \mathbf{R}$.

(c) Let $z = 2 + 4i$.

(i) Express $z^2 + 28$ in the form $p + qi$ where $p, q \in \mathbf{R}$.

(ii) Solve for real k

$k(z^2+28) = |z|(1+i)$.

Express your answer in the form $\frac{\sqrt{a}}{b}$ where $a, b \in \mathbf{N}$ and a is a prime number.

SOLUTION

4 (a)

$7(2+i)+i(11+9i)$ [Multiply out the brackets.]

$=14+7i+11i+9i^2$ [Tidy up using the fact that $i^2 = -1$.]

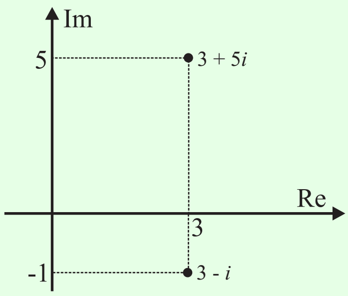
$=14+18i-9$

$=5+18i$

4 (b) (i)

$w = 3 - i$

$w + 6i = 3 - i + 6i = 3 + 5i$



4 (b) (ii)

Finding the modulus:

$z = a + bi \Rightarrow |z| = \sqrt{a^2+b^2}$ 2

$|w+6i| = |3+5i| = \sqrt{3^2+5^2}$

$= \sqrt{9+25} = \sqrt{41}$

4 (b) (iii)

Working out the conjugate: $z = a+bi \Rightarrow \bar{z} = a-bi$ 1

Division: Multiply above and below by the conjugate of the bottom.

$\frac{1}{w+6i} = \frac{1}{3+5i}$ [Multiply above and below by the conjugate of the bottom.]

$= \frac{1}{(3+5i)} \times \frac{(3-5i)}{(3-5i)}$ [Multiply out the brackets.]

$= \frac{3-5i}{9-15i+15i-25i^2}$ [Tidy up using the fact that $i^2 = -1$.]

$= \frac{3-5i}{9+25} = \frac{3-5i}{34}$

$= \frac{3}{34} - \frac{5}{34}i$

4 (c) (i)

$z^2 + 28 = (2+4i)^2 + 28$

$= (2+4i)(2+4i) + 28$ [Multiply out the brackets.]

$= 4 + 8i + 8i + 16i^2 + 28$ [Tidy up using the fact that $i^2 = -1$.]

$= 32 + 16i - 16$

$= 16 + 16i$

4 (c) (ii)

For all equations you can equate (set equal) the real parts and the imaginary parts.

$k(z^2 + 28) = |z|(1+i)$

$\Rightarrow k(16+16i) = |2+4i|(1+i)$

$\Rightarrow 16k + 16ki = \sqrt{2^2+4^2}(1+i)$

$\Rightarrow 16k + 16ki = \sqrt{20}(1+i)$

$\Rightarrow 16k + 16ki = \sqrt{20} + \sqrt{20}i$

Finding the modulus:

$z = a+bi \Rightarrow |z| = \sqrt{a^2+b^2}$ 2

Equate the real parts: $16k = \sqrt{20} \Rightarrow k = \frac{\sqrt{20}}{16} = \frac{2\sqrt{5}}{16} = \frac{\sqrt{5}}{8}$