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Complex Numbers (Q 4, Paper 1)

1999

4 (a) Let $z = 5 + 4i$, where $i^2 = -1$.
Plot
(i) z

(ii) $z - 4i$
on an Argand diagram.

(b) Let $u = 3 - 6i$.
(i) Calculate $|u|$.

(ii) Show that $iu + \frac{u}{i} = 0$.

(iii) Express $\frac{u}{u+3i}$ in the form $p + qi$, $p, q \in \mathbf{R}$.

(c) Let $w = i - 2$.
Express w^2 in the form $a + bi$, $a, b \in \mathbf{R}$.
Hence, solve

$$kw^2 = 2w + 1 + ti$$

for real k and real t.

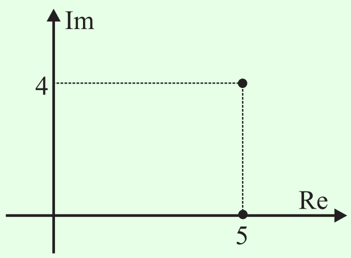
Solution

4 (a) (i)

$z = 5 + 4i$

4 (a) (ii)

$z - 4i = 5 + 4i - 4i = 5 + 0i$



4 (b) (i)

Finding the modulus:

$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$ 2

$|u| = |3 - 6i| = \sqrt{3^2 + (-6)^2}$
$= \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$

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4 (b) (ii)

$$iu+\frac{u}{i}=i(3-6i)+\frac{3-6i}{i}$$

$$=3i-6i^2+\frac{(3-6i)}{i}\times\frac{i}{i}$$ [If i is on the bottom of a fraction, multiply above and below by i.]

$$=3i+6+\frac{3i-6i^2}{i^2}$$ [Tidy up using the fact that $i^2=-1$.]

$$=3i+6+\frac{3i+6}{-1}$$

$$=3i+6-3i-6$$

$$=0$$

4 (b) (iii)

Working out the conjugate: $z=a+bi\Rightarrow\bar{z}=a-bi$ **1**

Division: Multiply above and below by the conjugate of the bottom.

$$\frac{u}{u+3i}=\frac{3-6i}{3-6i+3i}$$

$$=\frac{3-6i}{3-3i}$$ [Divide each term above and below by 3.]

$$=\frac{1-2i}{1-i}$$ [Multiply above and below by the conjugate of the bottom.]

$$=\frac{(1-2i)}{(1-i)}\times\frac{(1+i)}{(1+i)}$$ [Multiply out the brackets.]

$$=\frac{1+i-2i-2i^2}{1+i-i-i^2}$$ [Tidy up using the fact that $i^2=-1$.]

$$=\frac{1-i+2}{1+1}=\frac{3-i}{2}$$ [Divide the 2 on the bottom into each term above.]

$$=\tfrac{3}{2}-\tfrac{1}{2}i$$

4 (c)

$$w=i-2\Rightarrow w^2=(i-2)^2=(i-2)(i-2)$$ [Multiply out the brackets.]

$$=i^2-2i-2i+4$$ [Tidy up using the fact that $i^2=-1$.]

$$=-1-4i+4$$

$$=3-4i$$

For all equations you can equate (set equal) the real parts and the imaginary parts.

$$kw^2=2w+1+ti$$

$$\Rightarrow k(3-4i)=2(i-2)+1+ti$$

$$\Rightarrow 3k-4ki=2i-4+1+ti$$

$$\Rightarrow 3k-4ki=-3+(t+2)i$$ [Gather up the real parts and the imaginary parts.]

Equate the real parts: $3k=-3\Rightarrow k=-1$

Equate the imaginary parts: $-4k=t+2\Rightarrow -4(-1)=t+2\Rightarrow 4=t+2\Rightarrow t=2$