COMPLEX NUMBERS (Q 4, PAPER 1)

1997

4 (a) Simplify

$3(1+5i)+i(3-2i)$

and express your answer in the form $p + qi$, where $p, q \in \mathbf{R}$ and $i^2 = -1$.

(b) (i) For what values of a is

$|a+8i| = 10$ where $a \in \mathbf{R}$?

(ii) If $w = 4i$, verify that

$w^3 - w^2 + 16w - 16 = 0.$

(c) Let $z = 1 + i$ and let $\bar{z}$ be the complex conjugate of z.

Express $\frac{z}{\bar{z}}$ in the form $x + yi$, $x, y \in \mathbf{R}$.

Hence solve $k\left(\frac{z}{\bar{z}}\right) + tz = -3 - 4i$

for real k and t.

SOLUTION

4 (a)

$3(1+5i)+i(3-2i)$ [Multiply out the brackets.]

$= 3 + 15i + 3i - 2i^2$ [Tidy up using the fact that $i^2 = -1$.]

$= 3 + 18i + 2$ [Add the real numbers together and the imaginary numbers together.]

$= 5 + 18i$

4 (b) (i)

For all equations you can equate (set equal) the real parts and the imaginary parts.

Finding the modulus:

$z = a + bi \Rightarrow |z| = \sqrt{a^2 + b^2}$ **2**

$|a + 8i| = 10$

$\Rightarrow \sqrt{a^2 + 8^2} = 10$ [Square both sides.]

$\Rightarrow a^2 + 64 = 100$

$\Rightarrow a^2 = 100 - 64 = 36$

$\Rightarrow a = \pm\sqrt{36} = \pm 6$

4 (b) (ii)

$w^3 - w^2 + 16w - 16$
$= (4i)^3 - (4i)^2 + 16(4i) - 16$
$= 64i^3 - 16i^2 + 64i - 16$
$= -64i + 16 + 64i - 16$
$= 0$

Powers of i
$i = \sqrt{-1} = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

4 (c)

Working out the conjugate: $z = a + bi \Rightarrow \bar{z} = a - bi$ 1

Division: Multiply above and below by the conjugate of the bottom.

$z = 1 + i \Rightarrow \bar{z} = 1 - i$

$\frac{z}{\bar{z}} = \frac{1+i}{1-i}$ [Multiply above and below by the conjugate of the bottom.]

$= \frac{(1+i)}{(1-i)} \times \frac{(1+i)}{(1+i)}$ [Multiply out the brackets.]

$= \frac{1+i+i+i^2}{1+i-i-i^2}$ [Tidy up using the fact that $i^2 = -1$.]

$= \frac{1+2i-1}{1+1} = \frac{2i}{2}$

$= 1 = 1 + 0i$

For all equations you can equate (set equal) the real parts and the imaginary parts.

$k\left(\frac{z}{\bar{z}}\right) + tz = -3 - 4i$

$\Rightarrow k(1) + t(1+i) = -3 - 4i$

$\Rightarrow k + t + ti = -3 - 4i$

$\Rightarrow (k+t) + ti = -3 - 4i$ [Gather up the real parts and the imaginary parts.]

Equate the imaginary parts: $t = -4$
Equate the real parts: $k + t = -3 \Rightarrow k - 4 = -3 \Rightarrow k = 1$