COMPLEX NUMBERS (Q 4, PAPER 1)

LESSON NO. 6: EQUATIONS I

2007

4 (b) Let $z = 5 - 3i$.

(i) Plot z and $-z$ on an Argand diagram.

(ii) Calculate $|z - 1|$.

(iii) Find the value of the real number k such that $ki + 4z = 20$.

2006

4 (c) (i) Express $\dfrac{3-2i}{1-4i}$ in the form $x + yi$.

(ii) Hence, or otherwise, find the values of the real numbers p and q such that

$$p + 2qi = \frac{17(3-2i)}{1-4i}.$$

2005

4 (c) Let $z = 1 - 2i$.

(i) Write down $\bar{z}$, the complex conjugate of z.

(ii) Find the real numbers k and t such that

$$kz + t\bar{z} = 2z^2.$$

2004

4 (c) Let $z_1 = 5 + 12i$ and $z_2 = 2 - 3i$.

(i) Find the value of the real number k such that $|z_1| = k|z_2|$.

(ii) p and q are real numbers such that

$$\frac{z_1}{z_2} = p(q + i).$$

Find the value of p and the value of q.

2003

4 (c) Let $w = 1 + i$.

(i) Simplify $\frac{6}{w}$.

(ii) a and b are real numbers such that

$$a\left(\frac{6}{w}\right) - b(w+1) = 3(w+i).$$

Find the value of a and the value of b.

2001

4 (b) Solve

$$(x + 2yi)(1 - i) = 7 + 5i$$

for real x and for real y.

2000

4 (c) Let $z = 2 + 4i$.

(i) Express $z^2 + 28$ in the form $p + qi$ where $p, q \in \mathbf{R}$.

(ii) Solve for real k

$$k(z^2 + 28) = |z|(1 + i).$$

Express your answer in the form $\frac{\sqrt{a}}{b}$ where $a, b \in \mathbf{N}$ and a is a prime number.

1999

4 (c) Let $w = i - 2$.

Express w^2 in the form $a + bi$, $a, b \in \mathbf{R}$.

Hence, solve

$$kw^2 = 2w + 1 + ti$$

for real k and real t.

1998

4 (c) Let $u = 2 - i$.

(i) Express $u + \frac{1}{u}$ in the form $a + bi$, $a, b \in \mathbf{R}$.

(ii) Hence, solve

$$k(u + \tfrac{1}{u}) + ti = 18$$

for real k and real t.

1997

4 (c) Let $z = 1 + i$ and let $\overline{z}$ be the complex conjugate of z.

Express $\dfrac{z}{\overline{z}}$ in the form $x + yi, \quad x, y \in \mathbf{R}$.

Hence solve $k\left(\dfrac{z}{\overline{z}}\right) + tz = -3 - 4i$

for real k and t.

1996

4 (b) Let $w = (1 - 3i)(2 + i)$.

Express w in the form $p + qi, \quad p, q \in \mathbf{R}$.

Verify that

$$|w + \overline{w}| = |w - \overline{w}|,$$

where $\overline{w}$ is the complex conjugate of w.

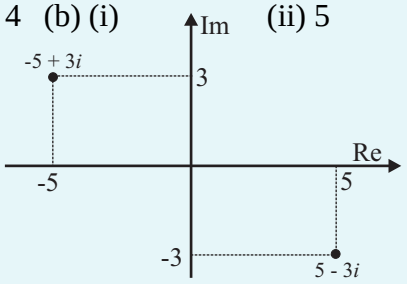
For what value of a is

$$\frac{\overline{w}}{2i} = aw,$$

where $a \in \mathbf{R}$?

ANSWERS

2007 4 (b) (i)



(ii) 5 (iii) $k = 12$

2006 4 (c) (i) $\frac{11}{17} + \frac{10}{17}i$ (ii) $p = 11, q = 5$

2005 4 (c) (i) $\overline{z} = 1 + 2i$ (ii) $t = -5, k = -1$

2004 4 (c) (i) $k = \sqrt{13}$ (ii) $p = 3, q = -\frac{2}{3}$

2003 4 (c) (i) $3 - 3i$ (ii) $a = -1, b = -3$

2001 4 (b) $x = 1, y = 3$

2000 4 (c) (i) $16 + 16i$ (ii) $k = \dfrac{\sqrt{5}}{8}$

1999 4 (c) $w^2 = 3 - 4i; k = -1, t = 2$

1998 4 (c) (i) $\frac{12}{5} - \frac{4}{5}i$ (ii) $k = \frac{15}{2}, t = 6$

1997 4 (c) $\dfrac{z}{\overline{z}} = 0 + i; t = -3, k = 1$

1996 4 (b) $w = 5 - 5i, a = \frac{1}{2}$