THE CIRCLE (Q 3, PAPER 2)

LESSON NO. 4: LINES INTERSECTING CIRCLES

2007

3 (b) The line $x-3y=0$ intersects the circle $x^2+y^2=10$ at the points a and b.

(i) Find the coordinates of a and the coordinates of b.

(ii) Show that $[ab]$ is a diameter of the circle.

SOLUTION

3 (b) (i)

1. $L: x-3y=0 \Rightarrow x=3y$

2. $C: x^2+y^2=10$

$\Rightarrow (3y)^2+y^2=10$

$\Rightarrow 9y^2+y^2=10$

$\Rightarrow 10y^2=10 \Rightarrow y^2=1$

$\therefore y=\sqrt{1}=1, -1$

$\therefore x=3, -3$

Points of intersection: $a(-3, -1)$, $b(3, 1)$

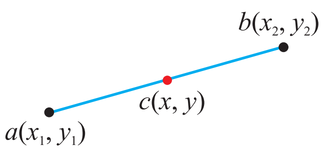
> **STEPS**
> **1.** Isolate x or y using equation of the line.
> **2.** Substitute into the equation of the circle and solve the resulting quadratic.

3 (b) (ii)

> **TO SHOW A LINE SEGMENT IS A DIAMETER OF A CIRCLE:**
> The midpoint of a diameter is the centre of a circle.

The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \ldots\ldots \; \textbf{2}$$



REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint} = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2}\right)$

$$\begin{array}{cccc} a(-1, & -3) & b(1, & 3) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\text{Midpoint of } [ab] = \left(\frac{-1+1}{2}, \frac{-3+3}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$$

Equation of Circle: $x^2+y^2=10$

Centre $(0, 0)$

> Circle C with centre $(0, 0)$, radius r.
> $x^2+y^2=r^2 \quad \ldots\ldots \; \textbf{1}$

Therefore, $[ab]$ is the diameter of the circle as its midpoint is the centre of the circle.

2005

3 (b) The line $y = 10 - 2x$ intersects the circle $x^2 + y^2 = 40$ at the points a and b.

(i) Find the coordinates of a and the co-ordinates of b.

(ii) Show the line, the circle and the points of intersection on a coordinate diagram.

SOLUTION

3 (b) (i)

STEPS
1. Isolate x or y using equation of the line.
2. Substitute into the equation of the circle and solve the resulting quadratic.

1. $L: y = 10 - 2x$

2. $C: x^2 + y^2 = 40$

$\Rightarrow x^2 + (10 - 2x)^2 = 40$

$\Rightarrow x^2 + 100 - 40x + 4x^2 = 40$

$\Rightarrow 5x^2 - 40x + 60 = 0$

$\Rightarrow x^2 - 8x + 12 = 0$

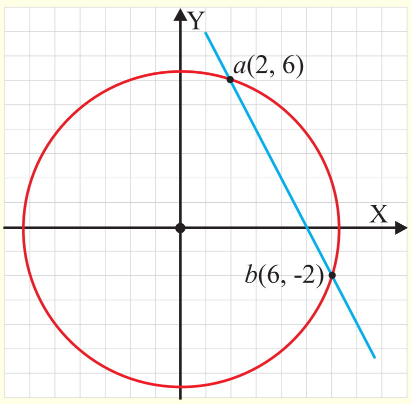
$\Rightarrow (x - 2)(x - 6) = 0$

$\therefore x = 2, 6$

$\therefore y = 6, -2$

Points of intersection: $a(2, 6)$, $b(6, -2)$

3 (b) (ii)



3 (c) (i)

Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2 + (y-k)^2 = r^2}$ 2

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

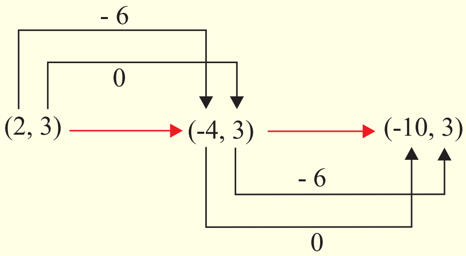
$K: (x+4)^2 + (y-3)^2 = 36$

Centre $(-4, 3)$, $r = \sqrt{36} = 6$

CONT....

3 (c) (ii)

Find the image of (2, 3) by a central symmetry through the centre $(-4, 3)$.



$$(2, 3) \to (-4, 3) \to (-10, 3)$$

3 (c) (iii)

As $(-4, y)$ is on the circle K, you can substitute it into the equation of K.

$(-4, y) \in (x+4)^2 + (y-3)^2 = 36$

$\Rightarrow (-4+4)^2 + (y-3)^2 = 36$

$\Rightarrow 0 + (y-3)^2 = 36$

$\Rightarrow (y-3)^2 = 36$

$\Rightarrow (y-3) = \pm 6$

$\therefore y = -3, 9$

2003

3 (b) The line $x-2y+5=0$ intersects the circle $x^2+y^2=10$ at the points *a* and *b*.

(i) Find the co-ordinates of *a* and the co-ordinates of *b*.

(ii) Draw a coordinate diagram on graph paper, showing the line, the circle and the points of intersection.

SOLUTION

3 (b) (i)

STEPS
1. Isolate *x* or *y* using equation of the line.
2. Substitute into the equation of the circle and solve the resulting quadratic.

1. $L: x-2y+5=0 \Rightarrow x=2y-5$

2. $C: x^2+y^2=10$

$\Rightarrow (2y-5)^2+y^2=10$

$\Rightarrow 4y^2-20y+25+y^2=10$

$\Rightarrow 5y^2-20y+15=0$

$\Rightarrow y^2-4y+3=0$

$\Rightarrow (y-1)(y-3)=0$

$\therefore y=1, 3$

$\therefore x=-3, 1$

Points of intersection: $a(-3, 1)$, $b(1, 3)$

3 (b) (ii)