THE CIRCLE (Q 3, PAPER 2)

LESSON NO. 2: THE HARDER CIRCLE

2005

3 (c) The circle K has equation $(x+4)^2+(y-3)^2=36$.

(i) Write down the coordinates of the centre of K.

(ii) The point (2, 3) is one end-point of a diameter of K.
Find the coordinates of the other end-point.

(iii) The point $(-4,\ y)$ is on the circle K. Find the two values of y.

SOLUTION

3 (c) (i)

Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2+(y-k)^2=r^2}$ 2

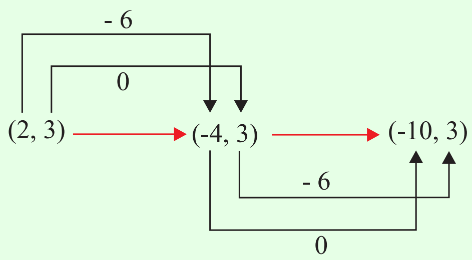
To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$K:(x+4)^2+(y-3)^2=36$

Centre $(-4, 3)$, $r=\sqrt{36}=6$

3 (c) (ii)

Find the image of (2, 3) by a central symmetry through the centre $(-4, 3)$.



$(2, 3)\rightarrow(-4, 3)\rightarrow(-10, 3)$

3 (c) (iii)

As $(-4,\ y)$ is on the circle K, you can substitute it into the equation of K.

$(-4,\ y)\in(x+4)^2+(y-3)^2=36$

$\Rightarrow(-4+4)^2+(y-3)^2=36$

$\Rightarrow 0+(y-3)^2=36$

$\Rightarrow(y-3)^2=36$

$\Rightarrow(y-3)=\pm 6$

$\therefore y=-3, 9$

2004

3 (c) K is a circle with centre $(-2, 1)$. It passes through the point $(-3, 4)$.

(i) Find the equation of K.

(ii) The point $(t, 2t)$ is on the circle K.
Find the two possible values of t.

SOLUTION

3 (c) (i)

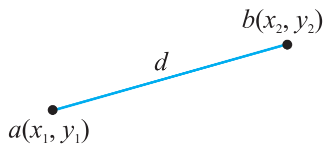
To form the equation of K you need its centre (given) and its radius (you can find the radius by working out the distance between the centre and the point on the circle).

$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$ **1**

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$



$$\begin{array}{cccc} (-2, & 1) & (-3, & 4) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$r = \sqrt{(-3-(-2))^2 + (4-1)^2}$

$\Rightarrow r = \sqrt{(-3+2)^2 + (4-1)^2}$

$\Rightarrow r = \sqrt{(-1)^2 + (3)^2} = \sqrt{1+9}$

$\therefore r = \sqrt{10}$

Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2 + (y-k)^2 = r^2}$ **2**

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

Centre $(-2, 1)$, $r = \sqrt{10}$

$K: (x+2)^2 + (y-1)^2 = (\sqrt{10})^2$

$\therefore (x+2)^2 + (y-1)^2 = 10$

3 (c) (ii)

It the point $(t, 2t)$ lies on K, then you can substitute into K.

$(t, 2t) \in (x+2)^2 + (y-1)^2 = 10$

$\Rightarrow (t+2)^2 + (2t-1)^2 = 10$

$\Rightarrow t^2 + 4t + 4 + 4t^2 - 4t + 1 = 10$

$\Rightarrow 5t^2 - 5 = 0 \Rightarrow 5t^2 = 5$

$\Rightarrow t^2 = 1$

$\therefore t = \sqrt{1} = \pm 1$

2001

3 (a) The circle S has equation $(x-3)^2+(y-4)^2=25$.

(i) Write down the centre and the radius of S.

(ii) The point $(k, 0)$ lies on S. Find the two real values of k.

SOLUTION

3 (a) (i)

Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2+(y-k)^2=r^2}$ **2**

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$S:(x-3)^2+(y-4)^2=25$

Centre $(3, 4)$, $r=\sqrt{25}=5$

3 (a) (ii)

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$(k, 0)\in S \Rightarrow (k-3)^2+(0-4)^2=25$

$\Rightarrow (k-3)^2+(-4)^2=25$

$\Rightarrow k^2-6k+9+16=25$

$\Rightarrow k^2-6k=0$

$\Rightarrow k(k-6)=0$

$\therefore k=0, 6$

1999

3 (b) The points $(-1, -1)$ and $(3, -3)$ are the end points of a diameter of a circle S.

(i) Find the coordinates of the centre of S.

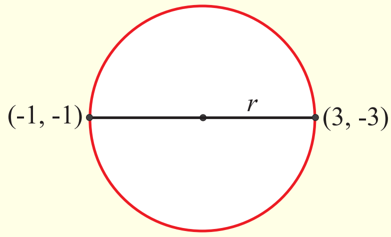
(ii) Find the radius length of S.

(iii) Find the equation of S.

SOLUTION

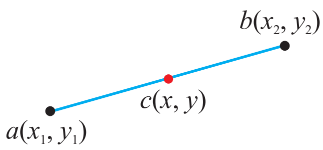
3 (b) (i)

The centre of the circle is the midpoint of the end points of the diameter.



The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\boxed{\text{Midpoint } = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)} \;\ldots\ldots\; \textbf{2}$$



REMEMBER THE MIDPOINT FORMULA AS: Midpoint $= \left(\dfrac{\text{Add the } x\text{'s}}{2}, \dfrac{\text{Add the } y\text{'s}}{2}\right)$

$$\begin{array}{cccc} (-1, & -1) & (3, & -3) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\text{Midpoint } = \left(\frac{-1+3}{2}, \frac{-1-3}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) = (1, -2)$$

3 (b) (ii)

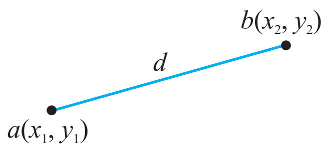
The radius is the distance from the centre $(1, -2)$ to either of the end points of the diameter, say $(-1, -1)$.

$$\boxed{d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}} \;\ldots\ldots\; \textbf{1}$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$$\begin{array}{cccc} (1, & -2) & (-1, & -1) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

$$r = \sqrt{(-1-1)^2 + (-1-(-2))^2}$$

$$\Rightarrow r = \sqrt{(-1-1)^2 + (-1+2)^2}$$

$$\Rightarrow r = \sqrt{(-2)^2 + (1)^2} = \sqrt{4+1}$$

$$\therefore r = \sqrt{5}$$

CONT....

3 (b) (iii)

Equation of S: centre $(h,\ k)=(1,\ -2),\ r=\sqrt{5}$

Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2+(y-k)^2=r^2}$ **2**

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$S:(x-h)^2+(y-k)^2=r^2$

$\Rightarrow (x-1)^2+(y-(-2))^2=(\sqrt{5})^2$

$\therefore (x-1)^2+(y+2)^2=5$

1997

3 (c) C is the circle with centre $(-1,\ 2)$ and radius 5.
Write down the equation of C.

The circle K has equation $(x-8)^2+(y-14)^2=100$.
Prove that the point $p(2, 6)$ is on C and on K.
Show that p lies on the line which joins the centres of the two circles.

SOLUTION

Equation of C: centre $(h,\ k)=(-1,\ 2),\ r=5$

$(x-h)^2+(y-k)^2=r^2$

$\Rightarrow (x-(-1))^2+(y-2)^2=(5)^2$

$\therefore (x+1)^2+(y-2)^2=25$

IS A POINT ON A CIRCLE, INSIDE A CIRCLE OR OUTSIDE A CIRCLE?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$C:(x+1)^2+(y-2)^2=25$

$p(2,\ 6)\in C$?

$(2+1)^2+(6-2)^2=(3)^2+(4)^2$

$=9+16=25 \Rightarrow p(2,\ 6)\in C$

$K:(x-8)^2+(y-14)^2=100$

$p(2,\ 6)\in K$?

$(2-8)^2+(6-14)^2=(-6)^2+(8)^2$

$=36+64=100 \Rightarrow p(2,\ 6)\in K$

CONT....

Write down the centres of C and K.

Circle C with centre (h, k), radius r.

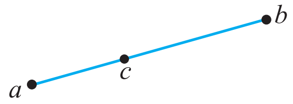
$$\boxed{(x-h)^2+(y-k)^2=r^2} \quad \ldots\ldots\ 2$$

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$$C:(x+1)^2+(y-2)^2=25 \Rightarrow \text{centre } p_1(-1, 2)$$

$$K:(x-8)^2+(y-14)^2=100 \Rightarrow \text{centre } p_2(8, 14)$$

Collinear points: Three points are collinear if the slope of any two points equals the slope of any other two points.
Ex. a, b and c are collinear if you can show that:
Slope of ac = Slope of cb



To show all three points are on the same line (collinear), show that the slope of pp_1 equals the slope of pp_2.

$$\boxed{m=\frac{y_2-y_1}{x_2-x_1}} \quad \ldots\ldots\ 3$$

Remember it as:

$$\text{Slope } m=\frac{\text{Difference in } y\text{'s}}{\text{Difference in } x\text{'s}}$$

$$\begin{array}{cccc} p(2, & 6) & p_1(-1, & 2) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of pp_1: $m_1=\dfrac{2-6}{-1-2}=\dfrac{-4}{-3}=\dfrac{4}{3}$

$$\begin{array}{cccc} p(2, & 6) & p_2(8, & 14) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}$$

Slope of pp_2: $m_2=\dfrac{14-6}{8-2}=\dfrac{8}{6}=\dfrac{4}{3}$

Therefore, p lies on the line which joins the centres of the two circles.

1996

3 (b) The points (1, 0) and (4, 4) are the end points of a diameter of a circle C.

(i) Find the coordinates of the centre of C.

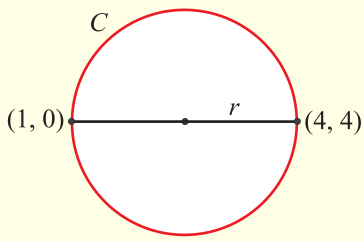
(ii) Find the radius length of C.

(iii) Find the equation of C.

SOLUTION

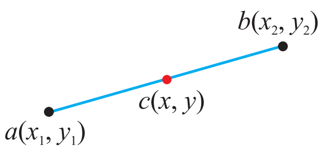
3 (b) (i)

The centre of the circle is the midpoint of the end points of the diameter.



The formula for the midpoint, c, of the line segment $[ab]$ is:

$$\boxed{\text{Midpoint } = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)} \text{ .... } 2$$



REMEMBER THE MIDPOINT FORMULA AS: $\text{Midpoint } = \left(\frac{\text{Add the } x\text{'s}}{2}, \frac{\text{Add the } y\text{'s}}{2}\right)$

$$\begin{array}{cc} (1, 0) & (4, 4) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$$\text{Midpoint (centre) } = \left(\frac{1+4}{2}, \frac{0+4}{2}\right) = \left(\frac{5}{2}, \frac{4}{2}\right) = (\tfrac{5}{2}, 2)$$

3 (b) (ii)

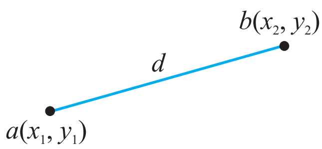
The radius is the distance from the centre $(\tfrac{5}{2}, 2)$ to either of the end points of the diameter, say (1, 0)

$$\boxed{d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}} \text{ .... } 1$$

The distance between a and b is written as $|ab|$.

REMEMBER THE DISTANCE FORMULA AS:

$$d = \sqrt{(\text{Difference in } x\text{'s})^2 + (\text{Difference in } y\text{'s})^2}$$



$$\begin{array}{cc} (\tfrac{5}{2}, 2) & (1, 0) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

$$r = \sqrt{(1-\tfrac{5}{2})^2+(0-2)^2}$$

$$\Rightarrow r = \sqrt{(-\tfrac{3}{2})^2+(-2)^2}$$

$$\Rightarrow r = \sqrt{\tfrac{9}{4}+4} = \sqrt{\tfrac{9}{4}+\tfrac{16}{4}}$$

$$\therefore r = \sqrt{\tfrac{25}{4}} = \tfrac{5}{2}$$

CONT....

3 (b) (iii)

Equation of C: centre $(h,\ k) = (\frac{5}{2},\ 2)$, $r = \frac{5}{2}$

Circle C with centre (h, k), radius r.

$\boxed{(x-h)^2 + (y-k)^2 = r^2}$ 2

To get the centre: Change the sign of the number inside each bracket.
To get the radius: Take the square root of the number on the right.

$C: (x-h)^2 + (y-k)^2 = r^2$

$\Rightarrow (x-\frac{5}{2})^2 + (y-2)^2 = (\frac{5}{2})^2$

$\therefore (x-\frac{5}{2})^2 + (y-2)^2 = \frac{25}{4}$